Dynamics of wind turbines

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Abstract: Three principal aspects of the dynamics of wind-turbine behaviour are discussed: forced response to deterministic loads, forced response to stochastic loads and stability. In each case, an introduction to the physics of the problem is presented and a means of analysis is described. A simple analytical model is derived to help illustrate some of the analytical techniques commonly employed.

1 Introduction

The most noticeable change in wind-turbine design, during the last ten years, is in the increased flexibility of the structure as a whole. This change is well illustrated by comparison of the NASA MOD-0 and the Boeing MOD-2 machines. The most apparent outward sign of this evolution is the change in tower structure, the stiff-truss tower being replaced by a considerably softer cylinderical structure. The tower structure, although the most visible, is not the only component to evolve in this way. The same procedure has taken place with rotors (in some cases, fibre glass and wood are replacing steel) and with transmission systems where stiff, rugged versions are being replaced by flexible ones of various types, both mechanical and electrical. Almost all large two-bladed machines now have teetered hubs rather than rigid ones. There is, at present, no concensus of opinion about the ideal combination of structural components. The solution adopted depends on the size, duty and location of each machine. Opting for a flexible tower, for instance, does not necessarily imply the use of a flexible transmission system or rotor.

The increasing structural flexibility of wind turbines means that their dynamic behaviour, and our ability to predict it, becomes more important. Inherent with increased flexibility are large displacements which may give rise to large inertial loads and, in some cases, instabilities.

This paper reviews some problems that arise in wind-turbine dynamics and describes methods for their analysis. Only horizontal-axis machines will be treated in any detail. Vertical-axis machines have many similarities, but also many important differences which put their treatment outside the scope of this review.

2 Basic analysis

The dynamic problems that are encountered in wind-turbine systems may be conveniently divided into three separate sections, by means of identifying the different types of forcing functions that are involved. These categories are:

(*a*) stability
(*b*) forced response to deterministic loads
(*c*) forced response to stochastic loads.

Stability is a property of the system and may be determined by analysis of the homogeneous equations of motion. Stability analysis is, therefore, characterised by the absence of a forcing function. This is not strictly true, but this definition serves as a means of categorising the nature of the problem.

The rotor of a wind turbine operating in the atmosphere will encounter wind velocities that are continuously changing. These changes come from a number of sources. The wind turbine will be controlled to yaw itself into the wind, but there will inevitably be some error giving yaw misalignment, which results in one blade moving into the wind while the others move out of it. A similar effect results from shaft tilt and wind pitch. The presence of the earth's boundary layer means that the air near the ground has a lower velocity than that higher up, resulting in a velocity gradient across the rotor disc. When the wind flows round the tower, it is declerated so that, in the neighbourhood of the tower, there is an area of retarded air. All these sources combine together to give a fairly complicated cyclic variation in wind speed and, hence, load, as the blade rotates; although these loads may be complicated, they are well defined and, hence, are termed 'deterministic'. In addition to this cyclic variation, there will be changes in wind speed resulting from wind turbulence which produces stochastic loads. It is useful to make a distinction between the cyclic, deterministic and the turbulent, stochastic variations.

The deterministic loads may be calculated relatively easily. Detailed discussion of the process is, however, outside the scope of this article. Most authors adopt a pseudo-steady-state approach to the aerodynamic calculations, the basic principles of which are given in Wilson and Lissaman [1]. This approach is not strictly applicable to cyclic conditions and some consideration is given to proper means of dealing with the periodic nature of the loads by Miller *et al.* [2]. The cyclic nature of the loads also suggests that unsteady effects such as stall hysteresis may be important; so far this has received little attention for HAWTs, although some work is presently under way; see, for example, Reference 3. A recent review of horizontal-axis wind-turbine aerodynamics may be found in de Vries [4, 5].

Assuming that adequate means are available for the determination of the cyclic aerodynamic loads, we may consider the most basic type of dynamic analysis that is available for any form of rotating machinery. The underlying principle involved in designing a turbine against the deterministic loads is the avoidance of resonance. This topic was the subject of a recent review paper by Sullivan [6]. The basic, and indeed essential, means of avoiding the coincidence of a frequency corresponding to the natural mode of the structure and that of a forcing function is the ability to perform reliable calculations of the natural frequencies of the important modes of vibration of the structure. Nowadays, most designers have finite-element packages available to them for such calculations, and no

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pioneering work is required for their application to stationary wind-turbine systems. The most convenient way of presenting and interpreting these results is in at interference diagram, sometimes called a Campbell diagram, an example of which is shown in Fig. 1. The finite-element

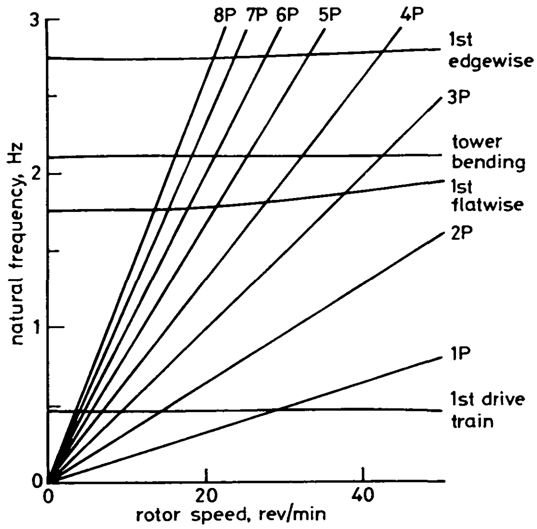

Fig. 1 *Typical interference diagram*

calculations are usually performed for a stationary system, although the frequencies of interest are those corresponding to the natural modes of vibration of the rotating system. Proper calculation of these is fairly complicated and will be dealt with in the following text; however, a reasonably accurate estimate may be obtained by simply allowing for the centrifugal stiffening effects on the rotating blades, which manifest themselves as a change in natural frequency with rotational speed, as shown in Fig. 1. The interference diagram shows the natural frequencies, together with the rotational speed and its harmonics. It is considered prudent to arrange these so that the harmonic rays do not coincide or come near to coinciding with the natural frequencies at the operating speed, thus avoiding possible resonances.

In general, each blade will see the cyclic aerodynamic loads at a frequency of once per revolution (or $1P$), and may also be affected by the harmonics. The importance of the harmonics depends on the particular load case considered. The support structure will experience the sum of the blade loads at any particular instant, and, if all the blades are identical, these loads will be modulated so that an n-bladed rotor will give rise to loads at a frequency of nP and its harmonics on the support structure. Small amplitude loads, at frequencies other than these, will arise because of slight imbalance between the blades. This may be due to manufacturing imperfections or difference in blade setting due to operation of the control system.

3 Dynamic modelling

Despite the fact that wind-turbine technology is relatively new, there are, in fact, quite a few mathematical models available which attempt to compute the aeroelastic behaviour of HAWTs. These vary from the very simple, such as the closed-form solution described by Stoddard [7] through limited degrees of freedom models, Garrad [8], to very complex systems, of which there are now quite a number; see, for example, Hoffmann [9], Friedmann and Warmbrodt [10], Vollan [11], Fabian [12] and Thurgood.†

Thurgood, D.A. (British Aerospace Dynamics Group, Hatfield, England) private communication

An excellent introduction to this subject may be found in the substantial text [13] developed by the Massachusetts Institute of Technology, which considers aerodynamics and transmission systems as well as general aeroelastic problems. A very good idea of the state of the art may be obtained from Reference 14. An overview of the subject is also presented by Thresher [15].

Any dynamic analysis of a wind turbine that addresses the problems of stability or forced response will rapidly encounter some common difficulties. Many engineers, faced with problems in structural dynamics, will immediately consider the use of one of the finite-element packages that are now very widely available. However, a particularly interesting aspect of dynamic modelling of a complete wind-turbine system is the fact that gross movement of one part of the structure occurs relative to another part. This precludes the use of standard finite-element packages that normally consider structures in which motion occurs about a mean undisplaced position. For this reason, all dynamic analysis packages used for wind turbines have had to be specially constructed. Some have opted for finite-element approaches which are developed from first principles [12], some for lumped parameter methods,† but most have opted for a modal description.

Whatever approach is used, similar problems and results are encountered. It is useful to consider a specific, but highly idealised, model to demonstrate some fairly general points. As an example, let us take the three degrees of freedom system shown in Fig. 2. This model has been

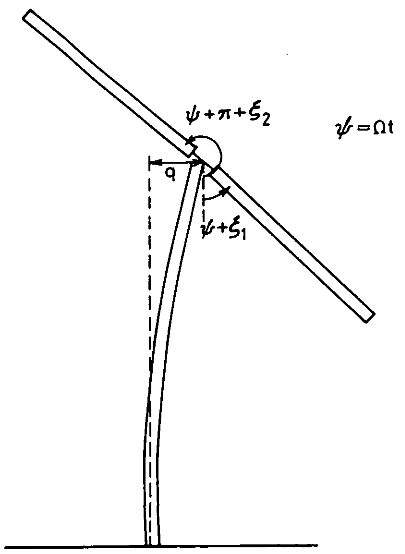

Fig. 2 *Simple three degrees of freedom model*

used in various other papers, see for example Dugundji *et al.* [13]; it is intended to represent a two-bladed rotor mounted on a flexible tower. The blades are allowed to exercise independent in-plane 'lead-lag' motion, and the tower is permitted to move laterally in the plane of rotation. If ξ_i represents the lead-lag angle of blade i, ψ the azimuth and q the linear motion of the tower head, then the kinetic energy of the system may be easily calculated to give:

$$T = \frac{1}{2}\left[I \sum_{i=1}^{2} (\dot{\psi}_i + \dot{\xi}_i)^2 + 2M\dot{q}^2 + 2\dot{q} \sum_{i=1}^{2} (\dot{\psi}_i + \dot{\xi}_i) \cos(\psi_i + \xi_i) + M_e \dot{q}^2 \right] \quad (1)$$

where

$$\psi_i = \psi + (i-1)\pi$$

$$I = \int_0^R mr^2\, dr$$

$$S = \int_0^R mr\, dr$$

$$M = \int_0^R m\, dr$$

M_e is the mass of the tower, R is the radius of the blades and m is their mass per unit length.

Assuming further that the blade and tower flexibilities may be expressed in terms of a simple spring, the potential energy may also be formulated as:

$$U = \tfrac{1}{2}K_\xi \sum_{i=1}^{2} \xi_i^2 + \tfrac{1}{2}K_t q^2 \tag{2}$$

where K_ξ represents an equivalent spring for the blades and K_t is a similar parameter for the tower.

Equations of motion for wind-turbine systems tend to be very clumsy, as they involve rotating and nonrotating components, and, hence, a well organised approach to their derivation is essential. Virtually all authors adopt the well known Lagrangian method that permits the derivation of equations of motion to be performed mechanically, after expressions for the kinetic and potential energy have been formed. The equation of motion for the generalised co-ordinate q_i is given by:

$$\frac{d}{dt}\frac{\partial T}{\partial \dot{q}_i} - \frac{\partial T}{\partial q_i} + \frac{\partial U}{\partial q_i} = Q_i \tag{3}$$

where Q_i is the generalised force. In this particular example, we intend to neglect all influences except the mechanical coupling in the system, so the generalised forces are zero and, hence, the equations of motion are:

$$\left.\begin{aligned} &M_t\ddot{q} + S\frac{d^2}{dt^2}[(\xi_1 - \xi_2)\cos\psi] + K_t q = 0 \\ &I\ddot{\xi}_1 + S\ddot{q}\cos(\psi + \xi_1) + K_\xi \xi_1 = 0 \\ &I\ddot{\xi}_2 - S\ddot{q}\cos(\psi + \xi_2) + K_\xi \xi_2 = 0 \end{aligned}\right\} \tag{4}$$

where $M_t = M_e + 2M$, the total linear mass moving at the top of the tower.

4 Stability

These equations may be linearised, which allows the term $\cos(\psi + \xi_i)$ to be reduced to $\cos\psi$, but the periodic terms cannot be removed. Thus, for a two-bladed rotor mounted on a flexible tower, the equations of motion written in matrix notation are always of the form:

$$[M(t)]\ddot{y} + [C(t)]\dot{y} + [K(t)]y = 0 \quad t = \psi/\Omega \tag{5}$$

For rotors with three or more blades, co-ordinate transformations exist that allow periodicity to be removed. However, as most large wind turbines have two blades, and a two-bladed rotor is more difficult to analyse, the discussion will be limited to that case.

There is, of course, no trouble in solving a linear differential equation of this sort, many numerical algorithms exist for that purposes. Indeed, in principle, the problem of solution would be no more complex for a realistic system than for the trivial example described here. It is highly desirable that the system is checked for stability. This can be done in the time domain by simply supplying a set of initial conditions and allowing the equations to be integrated over a long period of time. The solution may then be observed, to see if it is convergent or divergent. Such an approach is, however, unreliable, and a more direct method is desirable. Had the matrices in eqn. 5 been constant with time, standard eigenvalue analysis could be conducted to check for stability, and to determine the natural frequencies of the rotating system. The presence of the periodic coefficients do, however, preclude such an approach.

The problem of predicting the stability of differential equations with periodic coefficients is by no means new. A great deal of work has been done on the problem, largely inspired by the helicopter industry, where very similar dynamic problems occur. The wind-turbine community is, therefore, fortunate that it may apply the existing helicopter technology to wind-turbine problems. To illustrate the type of analysis that is required and to demonstrate the existence of at least one type of mechanical instability, we shall continue the analysis of the simple example described here.

The most widely used method is known as Floquet-Liapunov theory. Rather than describing the theory mathematically, it is perhaps useful to attempt to understand, in a general way, how the theory works. The first step of the approach is to transform the n equations of motion for an n degrees of freedom model into $2n$ first-order equations. Each state is individually perturbed by assigning it a unit initial value while all the other states have zero initial values. The system equations are then integrated around one revolution of the rotor, and the final solution vector $\boldsymbol{v}_1(T)$ is stored. This process is repeated for each state until $2n$ solution vectors $\boldsymbol{v}_1(T)$, $\boldsymbol{v}_2(T)$, ..., $\boldsymbol{v}_{2n}(T)$ have been obtained. These are assembled column by column into a matrix, termed the transition matrix $[Q]$. It seems sensible that, because such a matrix contains information about the transient behaviour, it should also be useful in analysing the stability of the system. It is outside the scope of this paper to demonstrate this fact, but knowledge of $[Q]$ is indeed the key to ascertaining the stability information about the system it describes.

λ_K are the eigenvalues of $[Q]$, such that $\lambda_K = e^{p_K t}$ and $p_K(= \alpha_K + i\omega_K)$ are the stability exponents of the system which may conveniently be determined by the relation:

$$\alpha_K = \frac{1}{T}\ln|\lambda_K|$$

and

$$\omega_K = \frac{1}{T}\tan^{-1}\left\{\frac{\mathrm{Im}(\lambda_K)}{\mathrm{Re}(\lambda_K)}\right\} \tag{6}$$

The damping in the system α_K is, therefore, uniquely determined. Unfortunately, ω_K is indeterminate by integer multiples of $2\pi/T = \Omega$. The actual frequency can either be determined by inspection or by obtaining a solution using the eigenvector corresponding to ω_K as an initial solution vector. The resulting solution will be periodic with frequency ω_K.

It should be apparent that use of Floquet theory is very cumbersome and expensive. It is necessary, when considering stability, to construct a root-locus plot, which means calculating the stability parameters for a range of rotational speeds. For the simple 3 dof system used here as an example, six rotations of the rotor must be performed to provide the transition matrix for a single eigenvalue calculation. For a more representative system with, say, twelve modal degrees of freedom, $2 \times 12 = 24$ rotations are required for each eigenvalue. Repeating this process for, say, ten rotational speeds, therefore, requires 240 revolutions. Some of the natural frequencies may be quite high and, hence, place considerable demands on the integration algorithm. It is, therefore, useful to consider more elegant approaches.

Physically it seems fairly obvious that the equations of motion themselves must contain the stability information, and it ought not, therefore, be necessary to solve them to deduce the system frequencies and damping. This approach has been persued by Kaza and Hammond [16], who have produced a considerably more economical scheme. Further information about Floquet analysis, in general, and wind-turbine stability analysis, in particular, may be found in References 10, 11, 13, 17 and 18.

Floquet analysis has been used to determine the system frequencies and damping of the model described by eqn. 4. The results of this analysis are presented in Fig. 3, which is

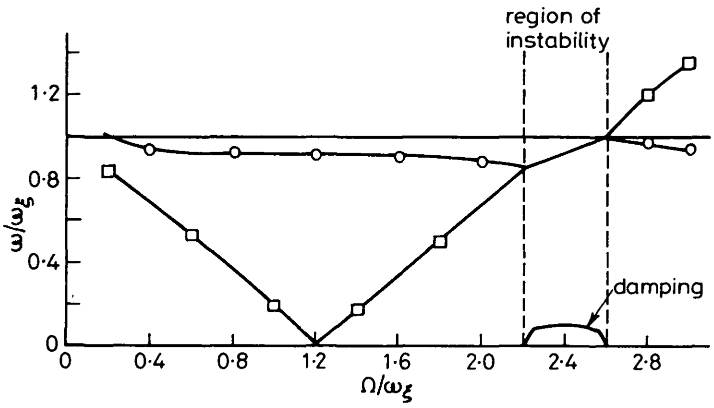

Fig. 3 *Typical stability plot*
——— rotor collective mode
—□— rotor cyclic mode
—○— tower lateral mode

typical of a stability plot for a wind-turbine system. Normally, the machine would operate at the low-frequency end of the plot. The frequencies here have been nondimensionalised with respect to $\sqrt{(K_\xi/I)} = \omega_\xi$. The important characteristic demonstrated by this plot is the coalescence of the tower mode and the blade lead-lag cyclic mode. In helicopter parlance, this coalescence represents an instability known as 'ground resonance' that can be very violent. Note also that, as the two modes coalesce, the damping which had hitherto been identically zero becomes finite, shown in the Figure as positive, for convenience, but is in fact destabilising. A model that included aerodynamics and structural damping would have had some finite but stabilising damping at all stable rotational speeds. It is customary to conduct stability analyses in the absence of structural damping, as its influence is very powerful, and it is reassuring to know that a system is stable without it.

In conclusion to this discussion of stability analysis, some comment should be made about the physical characteristics of mechanical instabilities. It is important to appreciate the difference between instabilities and resonances. To excite a resonance, the system must be forced at a certain frequency that coincides with a natural frequency of the structure. The presence of an instability, on the other hand, will result in some arbitary perturbation of a linear system growing without limit in the absence of any forcing. The above illustration of mechanical instability used 'ground resonance' as an example. Ground resonance involves the in-plane motion of the hub and blades. It occurs when the in-plane motion of the blades generates inertial loads which react with the tower, in such a way as to produce hub motion that further excites the blade lagging motion. For the present generation of wind turbines, this particular instability cannot occur. At present, wind-turbine instabilities are not categorised as rigorously as helicopter instabilities; although they do not have particular names, they can occur and should be checked for during the design process. Finally, it should be stressed that the example used for illustrative purposes here is a drastic oversimplification. Typical analyses of real systems have many degrees of freedom, and may exhibit not only pure mechanical instabilities such as described here, but also true aeroelastic instabilities akin to the well known 'flutter' problems encountered in aircraft. To predict aeroelastic instabilities, aerodynamic effects must of course be included.

5 Forced response to deterministic loads

A true aeroelastic model of a wind turbine must contain a structural dynamic model of the system including the power train and control system, as well as the blades and tower. All of these elements play their part in determining the behaviour of the system as a whole when it is excited by aerodynamic loads. There is, in principle, no difference in deriving the mathematical description of such a model from the simple derivation described in the preceding text; precisely the same steps would be followed. However, the analysis would now have to include the generalised forces, which would be derived from considering the aerodynamic loads.

The basis of the aerodynamic calculations is exactly that used for performance and static load prediction. It is, however, important to appreciate that the structural velocities of the turbine are now superimposed onto the wind speed and rotational velocities. The additional velocities modify the angle of attack of the blade section and, hence, change the blade loads as illustrated for flatwise motion in Fig. 4. It is the change in angle of attack that results in so called 'aerodynamic damping'. This effect is essential to the aeroelastic behaviour of the system; it implies that the formulation of the generalised forces is fairly complex algebraically, although, in principle, it is straightforward. Unsteady effects may also be included at this stage.

It is easy to discuss such a task in a few words, but experience of such a problem soon shows that the algebraic manipulation involved in the formulation of a good structural model can be truly formidable and run to hundreds of pages of calculations. The accuracy of such a model is limited only by the analyst's stamina and care! Short of providing a detailed derivation and presentation of an aeroelastic model, little more need be said about structural response predictions. Interested readers can consult References 7 and 8, for relatively simple examples, and References 10, 11 and 12, for more complex analyses.

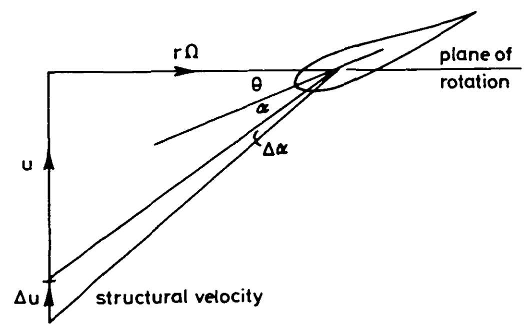

Fig. 4 *Impact of structural motion on angle of attack*
θ = built-in twist, α = angle of attack

Fig. 5 shows the blade root loads for the NASA MOD-0, which has been a workhorse for much experimental testing and validation of computer codes. Fig. 5*a* shows the flatwise bending moment as a function of azimuth. The dominant feature of this waveform is the large peak that occurs about 50° after bottom dead centre, which is a result of passing through the tower shadow. The MOD-0 rotor is downwind of the tower and hence this load is very large. For a HAWT, the dominant blade load for the inner portion, at least, is the gravitational bending moment which is shown clearly as a steady 1P oscillation in the edgewise loads of Fig. 5*b*. It is interesting to note the higher frequency oscillation in both of these loads. These are more prevalent in the edgewise than in the flatwise direction. The flatwise motion is heavily damped by virtue of the aerodynamics, whereas, even in a quite highly twisted blade such as this, the edgewise direction must rely mostly on structural damping, with a little help from the drive train. Upwind rotors will have more sinusoidal waveforms than these, but, otherwise, the characteristics will be similar. Any rotor which is subjected to an appreciable tower shadow, which is present in upwind as well as downwind rotors, will exhibit some high harmonic loads which result from the impulsive nature of the shadow. Fig. 5 also shows curves predicted by the author and reported by Garrad [8].

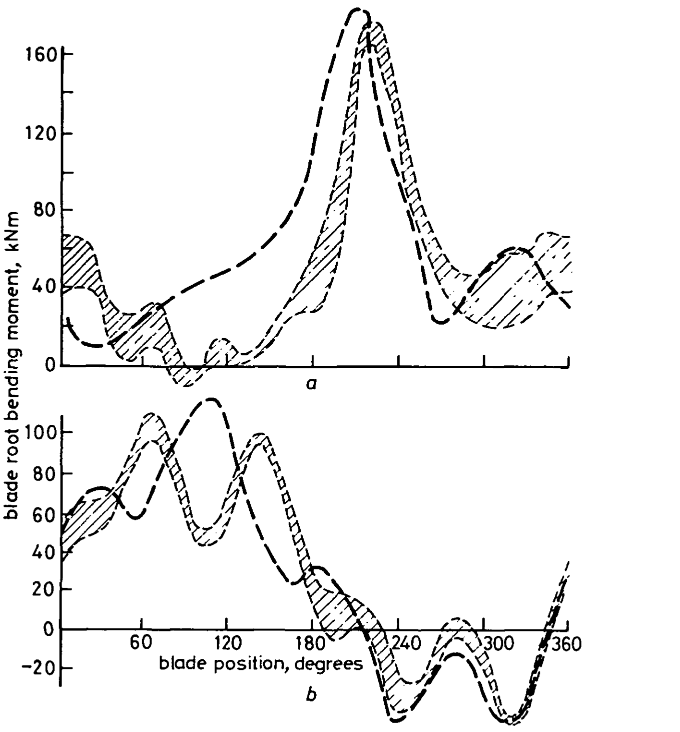

Fig. 5 *Mod-0 rigid hub loads—comparison of predicted and measured blade loads*
--- predicted, ▨ measured
a Flatwise
b Edgewise

The example cited was an early version of the MOD-0 machine which had a rigid hub. Later versions, and indeed most large two-bladed machines, have teetered rotors. For a rigid-hub machine, the dominant cyclic load on the shaft, nacelle and tower is the out-of-plane bending moment. Removal or reduction of this load helps considerably in the design of the components downwind of the rotor. This is achieved in two distinct ways, either by the addition of a third blade or by use of a teeter hinge. The addition of the third blade increases the rotor's symmetry so that the large periodic hub loads are greatly reduced by the spatial averaging of the three-bladed rotor. The addition of the teeter hinge, see Fig. 6, which permits the rotor to move as a rigid body out of the plane of rotation, allows the reaction to the out-of-plane moments to be derived from the considerable inertia of the rotor. The introduction of the teeter pin at the end of the shaft completely eliminates the troublesome moment loads.

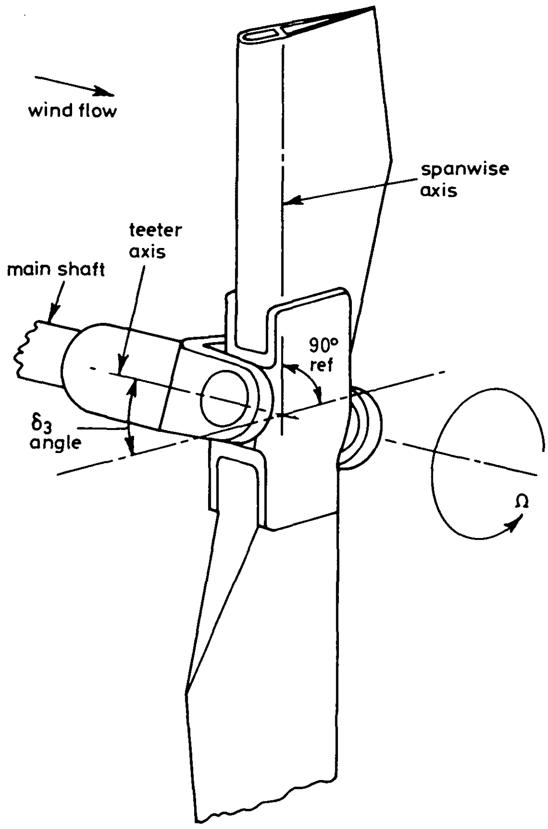

Fig. 6 *Teeter hinge*

Many different methods are available for predicting the types of loads described by Fig. 5; they differ quite radically in their complexity. In the USA and Europe, experimental data from large machines has been collected for about one year. Owing to the commercial nature of this information, little has been published. In the UK we are just entering the phase of comparing measurements with predicted dynamic loads, a process that should allow more reliable judgments about the efficiency of the various analytical procedures to be made.

6 Forced response to stochastic loads

To keep the analytical methods described in the preceding text in perspective, it is important to be sure that the various elements that make them up are of comparable accuracy. There seems little point in developing the structural or aerodynamic models too far, in the absence of an adequate representation of the behaviour of the wind itself; whose fluctuation is, after all, responsible for a large proportion of the load variation. Compared with the structural models, this aspect of wind-turbine analysis is very much in its infancy; although well established descriptions of the wind are available.

We have so far dealt with steady-state loading of rotors and their support structures. In addition to this cyclic loading, there will be transient loads that result from wind gusts, or, more accurately, from turbulent variations in the wind velocity. Wind turbines must be designed against these as well as steady-state loads. A fast acting control system may be able to alleviate these loads to some degree, but, as they result from a continuously varying source, such a course of action may result in considerable wear in the control system itself, and possibly in large loads in the rotor, if the control surfaces are made to move very fast. Whether the turbulent loads are removed or accommodated, it is necessary to be able to predict their nature. Such a step is significantly more complicated than the prediction of the deterministic loads.

The structural dynamic models used for prediction of stochastic loads are similar to those required for deterministic work, although it will normally be necessary to make some simplifications. The description of the wind does, however, require some discussion. The idea of a gust of wind is a familiar one. It is easy to understand and has the advantage of being relatively easy to analyse. Given the existence of a structural dynamic model of the system, using a time-domain integration scheme and bearing in mind the validity of any aerodynamic models used for local calculations, it is a fairly trivial step to perturb the wind input and, hence, model a gust. The transient behaviour of the system may be clearly observed with only minimal changes to any mathematical models adopted. There are plenty of data available on the modelling of these discrete gusts from the meteorological point of view. Frost [19] has compiled a fairly comprehensive set of data intended specifically for wind modelling for wind-turbine applications. The use of such a wind model has the advantage of analytical simplicity, but does not provide a realistic representation of the wind itself.

An important aspect of the natural wind is the distribution of turbulent energy at different frequencies, a characteristic that is easily described in the frequency domain. The modelling of the coherence, or rather incoherence, of the wind turbulence is also vital in providing a realistic input to load prediction procedures. The fact that, as the wind velocity increases on one part of the rotor disc, a corresponding increase does not necessarily occur elsewhere will obviously play an important part in determining differential loads. Both of these characteristics are difficult to model accurately using discrete gust methods.

It has long been recognised in the field of wind loading of stationary structures that frequency-domain methods are superior to discrete gust methods. There are well-established spectral representations of the wind: again these are conveniently collected by Frost [19] and a good state-of-the-art review is to be found in the CIRIA proceedings [20]. Hitherto, these methods have found little application in wind-turbine analysis. The reason for this is the added complexity introduced into the structure by the rotation of the blades. To understand the importance of this difference, a short digression is required.

It is useful to consider a turbulent eddy being convected past a structure. Consider, first, a stationary structure, as in Fig. 7*a*; if the eddy has a length l and the mean wind speed is u, a useful parameter is the time taken for this eddy to pass the building, which may be derived as $t_1 = l/u$. Now consider the same situation, except that the stationary structure is replaced by a rotating blade, Fig. 7*b*. For a point on the blade moving at a speed $r\Omega$, the time for the passage of the eddy is now the time taken for the blade to cut through it $t_2 = l/r\Omega$. The time is therefore considerably smaller for the turbine blade than for the stationary structure, the ratio being $u/r\Omega$, or the inverse of the local speed ratio $1/\lambda$. This process has the effect of moving the turbulent energy to higher frequencies and, in particular, to harmonics of the rotor speed; this is shown graphically in Fig. 8.

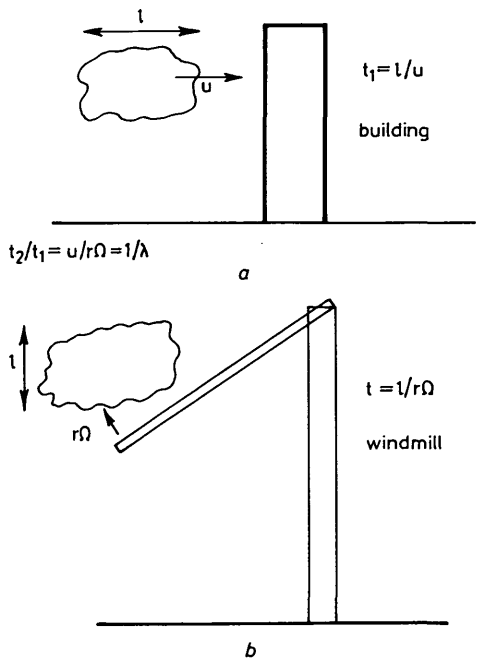

Fig. 7 *Turbulent loads*
a Stationary structure *b* Rotating turbine blade

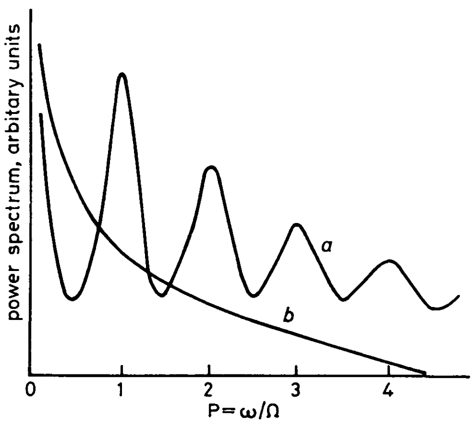

Fig. 8 *Turbulent velocity spectra seen from stationary and rotating frames of reference*
a Rotating frame, *b* stationary frame

The 'slicing' process may be conveniently described mathematically in terms of correlation functions. Many such functions exist and only one, the von Karman, will be used here. Assuming that the turbulence is homogeneous and that Taylor's 'frozen turbulence' hypothesis which relates spatial and temporal separation is valid, the cross-correlation of the velocity, at two points separated by the vector $\boldsymbol{r}$, may be expressed using the standard expression

$$\rho_{uu}(r, \tau) = \frac{2}{\Gamma(\frac{1}{3})}\left\{\left(\frac{\eta}{2}\right)^{1/3} K_{1/3}(\eta) - \frac{\pi}{4}\left(\frac{\Gamma(\frac{5}{6})}{\Gamma(\frac{1}{3})}\right)^2 \frac{r^2}{l^2}\left(\frac{\eta}{2}\right)^{-2/3} K_{-2/3}(\eta)\right\} \quad (7)$$

a detailed description of which may be found in Reference 20. $K_{1/3}$ and $K_{-2/3}$ are Bessel functions of the second kind, Γ is the Gamma function, l is the turbulence length scale,

$$\eta = \sqrt{\pi}\,\frac{\Gamma(\frac{5}{6})}{\Gamma(\frac{1}{3})}\frac{1}{l}(r^2 + u^2\tau^2)^{1/2} \quad (8)$$

and u is the mean wind speed. Simple physical reasoning shows that the wind that comes into contact with a point on a rotating wind-turbine blade occupies a spiral shape stretching upwind of the turbine, at any instant in time. This is in contrast to the straight line that would characterise the wind incident on a stationary building. By considering this spiral, eqn. 7 may be transformed to provide an expression for the crosscorrelation of the wind velocity fluctuations between two points on a wind-turbine rotor. This is achieved by replacing eqn. 8 with

$$\eta = \sqrt{\pi}\,\frac{\Gamma(\frac{5}{6})}{\Gamma(\frac{1}{3})}\,\frac{1}{l} \times \{r_1^2 + r_2^2 - 2r_1 r_2 \cos(\theta_s + \Omega\tau) + (u\tau)^2\}^{1/2} \quad (9)$$

where r_1 and r_2 are the radii of the two points considered, Ω is the speed of rotation and θ_s is an angle: $\theta_s = \pi$, if the points are on different blades and $\theta_s = 0$, if they are on the same blade. This particular transformation only applies for two-bladed machines.

This approach was first suggested by Rosenbrock [21] and has since been rediscovered by Kristensen and Frandsen [22] and Anderson [23]. It was validated experimentally by Connell [24]. A similar study is presented for helicopter rotors in Reference 25.

It is perhaps more familiar to consider turbulent fluctuations in spectral terms, rather than as correlation functions. Eqns. 7 and 9 lay the foundations for the use of a proper description of the wind, together with a structural model of the turbine. Such a calculation has been undertaken by Garrad and Hassan [26] and Madsen [27]. To do this in the frequency domain, the equations of motion must be transformed and generalised forces calculated by combining eqn. 7 with some suitable aerodynamic model of the blades. A description of this process is given by Garrad and Hassan [26]. In addition, some suitable usually fairly simple structural model must be incorporated. Using this approach, a response spectrum may be computed which has the rather clumsy form:

$$S_{jj}(p) = \text{constant} \times \frac{S_{jj}^F(p)}{|(P_j^2 - p^2 + iD_j p)|^2}$$

where

$$S_{jj}^F(p) = \int_{-1}^{1}\int_{-1}^{1} |x_K|\,|x_l| \times \phi_j(x_K)\phi_j(x_l)S^u(x_K, x_l, p)\,dx_K\,dx_l$$

p is nondimensional frequency $= \omega/\Omega$, D_j is a damping parameter dependent on the blade aerodynamics and mode shape, ϕ_j is the blade mode shape, x_K, x_l are nondimensional blade radii $= r/R$ and $S^u(x_K, x_l, p)$ is the Fourier transform of eqn. 7 with η taken from eqn. 9. Some typical response spectra for blade teetering and two-blade vibrational modes taken from Garrad and Hassan [26] are shown in Fig. 9. This Figure clearly demonstrates both the blade resonances and the peaks in the turbulent loads.

Use of a spectral model of the wind turbulence allows the coherence of the wind to be included in a realistic way. It also permits a proper description of frequency content of the wind to be included in the aeroelastic model. Use of such a model brings with it considerable analytical complexity. As the turbine rotor is by its very nature sensitive to changes in flow around it, coupling between the various modes of vibration of the rotor may occur via the aerodynamic loads. Application of spectral methods to stationary structures such as buildings usually assumes that the modes of vibration are uncoupled, because there is no feedback between the applied loads and resulting displacements. This will necessarily be the case, the modal independence being a basic building block of modal analysis.

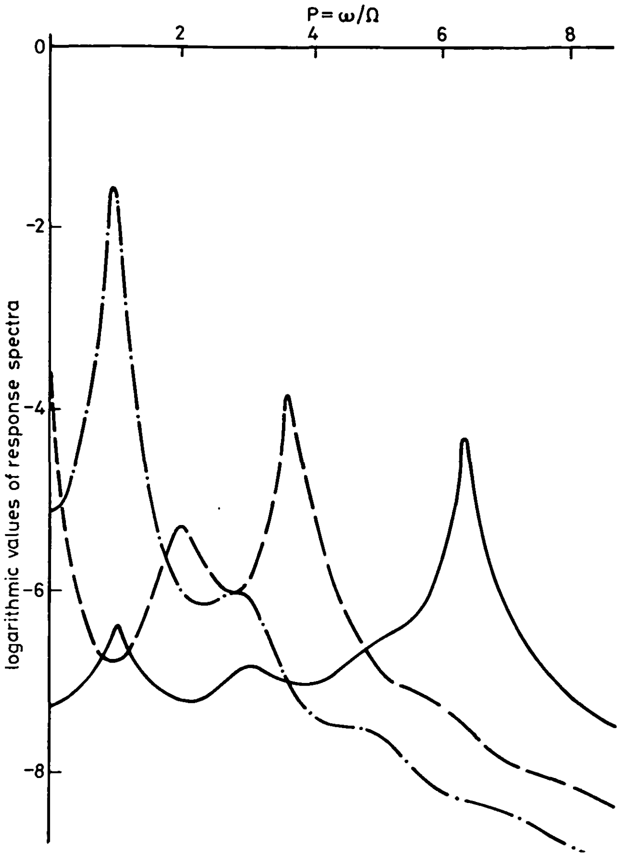

Fig. 9 *Structural response of a rotor to turbulent loads expressed as modal response in teeter, symmetric and asymmetric flatwise modes*
——— asymmetric, – – – symmetric, —·— teeter

The presence of coupling significantly complicates the analysis. The main aim of an analysis that includes wind turbulence will probably be the prediction of fatigue damage. It is not valid to combine the stochastic and deterministic loads by simple addition, and some further work is required to provide a suitable analytical basis for such a combination. The foundation for this work already exists in the field of communication engineering, see for example Bendat [28]. For rotors mounted on flexible support structures, it is important to include tower models in any dynamic analysis. For two-bladed rotors, the inclusion of a tower model and the consequent presence of periodic terms in the equations of motion, combined with the use of spectral methods, presents further complications.

Reference to Fig. 8 demonstrates that proper analysis of turbulent wind is required to estimate the stochastic loads. The Figure clearly demonstrates that, under certain conditions, the higher harmonic loads may assume considerable importance. The spatial distribution of the loads which can be accurately modelled by such a system is also of obvious importance.

It is evident from the experience of design teams, who have been operating large wind turbines, that the ability to predict fatigue life may rely very heavily on the availability of a dynamic model that contains a realistic wind description. No doubt in the near future the analytical effort in this field will increase substantially. Only spectral methods have been described here; there is also some research effort underway that attempts to simulate realistic turbulence in the time domain, using considerably more sophisticated methods than those usually described by the term 'discrete gust'.

7 Conclusions

This paper has attempted to outline some of the problems and means for their solution that occur in the dynamics of wind-turbine design. It was not intended to make a critical review of existing methods, but rather to enable an interested reader to obtain insight into the nature of the problems and to lead him to more detailed works where required.

The dynamics of rotating machines is a complex subject and, in the case of a wind turbine where the main motive force is so difficult to characterise, additional complications arise. The absence of any concensus of opinion about design choices, even over such a basic characteristic as the number of blades, demonstrates that there is still much to be learned about wind-turbine behaviour. This review has demonstrated that the dynamics of the turbine play a central role in design and, consequently, this aspect of wind-turbine technology will no doubt evolve considerably in the near future.

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