The following are derivations of the output motions available in FAST for a 2-bladed turbine configuration. The motions for a 3-bladed turbine are very similar. Note that some of the motions are given multiple names in order to support variation among the user's preferences.

Blade 1 Tip Motions: $OoPDefl = TipDxcl = [r^{QSl}(BldFlexL) - TipRadj_{3}^{Bl}] \cdot i_{1}^{Bl}$ Blade 1 OoP tip deflection (relative to rotor) (directed along the xc1-axis), (m) $IPDefl = TipDyc l = [r^{QSI}(BldFlexL) - TipRadj_{3}^{BI}] \cdot i_{2}^{BI}$ Blade 1 IP tip deflection (relative to rotor) (directed along the yc1-axis), (m) $TipDxbl = \left[r^{QSl} \left(BldFlexL \right) - TipRadj_{3}^{Bl} \right] \cdot j_{1}^{Bl}$ Blade 1 flapwise tip deflection (relative to rotor) (directed along the xb1-axis), (m) $TipDybl = \left[\mathbf{r}^{QSl} \left(BldFlexL \right) - TipRad\mathbf{j}_{3}^{Bl} \right] \cdot \mathbf{j}_{2}^{Bl}$ Blade 1 edgewise tip deflection (relative to rotor) (directed along the yb1-axis), (m) $TipDzcl = TipDzbl = \left\lceil r^{QSl} (BldFlexL) - TipRadj_{3}^{Bl} \right\rceil \cdot i_{3}^{Bl} = \left\lceil r^{QSl} (BldFlexL) - TipRadj_{3}^{Bl} \right\rceil \cdot j_{3}^{Bl}$ Blade 1 axial tip deflection (relative to rotor) (directed along the zc1-/zb1-axis). (m) $TipALxbl = {}^{E}a^{Sl} (BldFlexL) \cdot n_{1}^{Bl} (BldFlexL)$ Blade 1 flapwise tip acceleration (absolute) (directed along the xb1-axis), (m/sec²) $TipALybl = {}^{E}a^{Sl}(BldFlexL) \cdot n_{2}^{Bl}(BldFlexL)$ Blade 1 edgewise tip acceleration (absolute) (directed along the yb1-axis), (m/sec²) $TipALzbl = {}^{E}a^{Sl} (BldFlexL) \cdot n_{3}^{Bl} (BldFlexL)$ Blade 1 axial tip acceleration (absolute) (directed along the zc1-/zb1-axis), (m/sec²) $RollDefl1 = TipRDxb1 = \left(\frac{180}{\pi}\right)^{H} \theta^{MI} (BldFlexL) \cdot j_{1}^{BI}$ Blade 1 roll tip deflection (relative to the undeflected position), (about the xb1-axis), (deg) $PtchDefl = TipRDybl = \left(\frac{180}{\pi}\right)^{H} \boldsymbol{\theta}^{MI} (BldFlexL) \cdot \boldsymbol{j}_{2}^{BI}$ Blade 1 pitch tip deflection (relative to the undeflected position), (about the yb1-axis), (deg) where: ${}^{H}\boldsymbol{\theta}^{MI}(BldFlexL) = {}^{E}\boldsymbol{\omega}_{BIFI}^{MI}(BldFlexL)q_{BIFI} + {}^{E}\boldsymbol{\omega}_{BIFI}^{MI}(BldFlexL)q_{BIFI} + {}^{E}\boldsymbol{\omega}_{BIFI}^{MI}(BldFlexL)q_{BIFI}$

 $\begin{cases} \sqrt{\left[r^{os1}\left(BldFlexL\right)\cdot d_{1}\right]^{2} + \left[r^{os1}\left(BldFlexL\right)\cdot d_{2}\right]^{2} + \left[r^{os1}\left(BldFlexL\right)\cdot d_{3}\right]^{2}} & for r^{os1}\left(BldFlexL\right)\cdot d_{2} > 0 \\ \sqrt{\left[r^{os1}\left(BldFlexL\right)\cdot d_{1}\right]^{2} + \left[r^{os1}\left(BldFlexL\right)\cdot d_{3}\right]^{2}} & otherwise \end{cases}$ Blade 1 tip-to-tower clearance, (m) where: $r^{os1}\left(BldFlexL\right) = r^{ov} + r^{vP} + r^{PQ} + r^{os1}\left(BldFlexL\right)$

$\frac{\text{Blade 1 Local Span Motions:}}{SpniALxbl = {}^{E}a^{Sl}(R^{Spani}) \cdot n_{I}^{Bl}(R^{Spani})}$ $1,2,...,5), (\text{m/sec}^{2})$ $SpniALybl = {}^{E}a^{Sl}(R^{Spani}) \cdot n_{2}^{Bl}(R^{Spani})$ $1,2,...,5), (\text{m/sec}^{2})$ $SpniALzbl = {}^{E}a^{Sl}(R^{Spani}) \cdot n_{3}^{Bl}(R^{Spani})$ $1,2,...,5), (\text{m/sec}^{2})$

Blade 2 Tip Motions:

The output motions of blade 2 are similar to those of blade 1.

Blade Pitch Motions:

$$BldPitch1 = PtchPMzc1 = PtchPMzb1 = \left(\frac{180}{\pi}\right)BlPitch(1)$$

/minus zb1-axis), (deg)
$$BldPitch2 = PtchPMzc2 = PtchPMzb2 = \left(\frac{180}{\pi}\right)BlPitch(2)$$

Blade 1 local flapwise acceleration (absolute) of span station i (directed along the *local* xb1-axis) (i =

Blade 1 local edgewise acceleration (absolute) of span station i (directed along the *local* yb1-axis) (i =

Blade 1 axial acceleration (absolute) of span station *i* (directed along the zc1-/zb1-/local zb1-axis) (*i* =

Blade 1 pitch angle (position) (positive towards feather / about the minus zc1-

Blade 2 pitch angle (position) (positive towards feather / about the minus zc2-

/minus zb2-axis), (deg)

Teeter Motions:

$$TeetDefl = RotTeetP = TeetPya = \left(\frac{180}{\pi}\right)q_{Teet}$$
Rotor teeter angle (position) (about the ya-axis), (deg)

$$RotTeetV = TeetVya = \left(\frac{180}{\pi}\right)\dot{q}_{Teet}$$
Rotor teeter angular velocity (about the ya-axis), (deg/sec)

$$RotTeetA = TeetAya = \left(\frac{180}{\pi}\right)\ddot{q}_{Teet}$$
Rotor teeter angular acceleration (about the ya-axis), (deg/sec²)

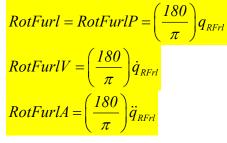
Shaft Motions:

$$Azimuth = LSSTipP = LSSTipPxa = LSSTipPxs = MOD\left[\left(\frac{180}{\pi}\right)(q_{DrTr} + q_{Gelt}) + AzimBIUp + 90,360\right]$$
Rotor azimuth angle (position) (about the xa-/xs-axis), (deg)
RotSpeed = LSSTipV = LSSTipVxa = LSSTipVxs = $\left(\frac{60}{2\pi}\right)(\dot{q}_{DrTr} + \dot{q}_{Gelt})$
Rotor azimuth speed / angular velocity (about the xa-/xs-axis), (rpm)
RotAccel = LSSTipA = LSSTipAxa = LSSTipAxs = $\left(\frac{180}{\pi}\right)(\ddot{q}_{DrTr} + \ddot{q}_{Gelt})$
Rotor azimuth angular acceleration (about the xa-/xs-axis), (deg/sec²)
LSSGagP = LSSGagPxa = LSSGagPxs = MOD $\left[\left(\frac{180}{\pi}\right)q_{Gelt} + AzimBIUp + 90,360\right]$
Low-speed shaft strain gage azimuth angle (position) (on the gearbox side of the low-speed shaft) (about the xa-/xs-axis), (deg)
LSSGagV = LSSGagVxa = LSSGagVxs = $\left(\frac{60}{2\pi}\right)\dot{q}_{Gelt}$
Low-speed shaft strain gage angular velocity (on the gearbox side of the low-speed shaft)
about the xa-/xs-axis), (rpm)
LSSGagA = LSSGagAxa = LSSGagAxs = $\left(\frac{180}{\pi}\right)\ddot{q}_{Gelt}$
Low-speed shaft strain gage angular acceleration (on the gearbox side of the low-speed shaft)
about the xa-/xs-axis), (deg/sec²)
HSShftV = $\left(\frac{60}{2\pi}\right)GBRatio \cdot \dot{q}_{Gelt}$
High-speed shaft speed / angular velocity, (rpm)
HSShftA = $\left(\frac{180}{\pi}\right)GBRatio \cdot \dot{q}_{Gelt}$
Tip speed ratio, (-)

Nacelle IMU Motions:

 $NcIMUTVxs = {}^{E}v^{IMU} \cdot c_{1}$ $NcIMUTVys = -{}^{E}v^{IMU} \cdot c_{3}$ $NcIMUTVzs = {}^{E}v^{IMU} \cdot c$ $NcIMUTAxs = {}^{E}a^{IMU} \cdot c_{I}$ $NcIMUTAys = -{}^{E}a^{IMU} \cdot c_{a}$ $NcIMUTAzs = {}^{E}a^{IMU} \cdot c$ $NcIMURVxs = \left(\frac{180}{\pi}\right)^{E} \boldsymbol{\omega}^{R} \cdot \boldsymbol{c}_{I}$ $NcIMURVys = -\left(\frac{180}{\pi}\right)^{E} \boldsymbol{\omega}^{R} \cdot \boldsymbol{c}_{3}$ $NcIMURVzs = \left(\frac{180}{\pi}\right)^{E} \boldsymbol{\omega}^{R} \cdot \boldsymbol{c}_{2}$ $NcIMURAxs = \left(\frac{180}{\pi}\right)^{E} \boldsymbol{\alpha}^{R} \cdot \boldsymbol{c}_{I}$ $NcIMURAys = -\left(\frac{180}{\pi}\right)^{E} \boldsymbol{\alpha}^{R} \cdot \boldsymbol{c}_{3}$ $NcIMURAzs = \left(\frac{180}{\pi}\right)^{E} \boldsymbol{\alpha}^{R} \cdot \boldsymbol{c}_{2}$

Rotor-Furl Motions:



Nacelle IMU translational velocity (directed along the xs-axis), (m/sec) Nacelle IMU translational velocity (directed along the ys-axis), (m/sec) Nacelle IMU translational velocity (directed along the zs-axis), (m/sec) Nacelle IMU translational acceleration (directed along the xs-axis), (m/sec²) Nacelle IMU translational acceleration (directed along the ys-axis), (m/sec²) Nacelle IMU translational acceleration (directed along the zs-axis), (m/sec²) Nacelle IMU angular (rotational) velocity (about the xs-axis), (deg/sec) Nacelle IMU angular (rotational) velocity (about the ys-axis), (deg/sec) Nacelle IMU angular (rotational) velocity (about the zs-axis), (deg/sec) Nacelle IMU angular (rotational) acceleration (about the xs-axis), (deg/sec²) Nacelle IMU angular (rotational) acceleration (about the ys-axis), (deg/sec²) Nacelle IMU angular (rotational) acceleration (about the zs-axis), (deg/sec²)

Rotor-furl angle (position) (about the rotor-furl axis), (deg) Rotor-furl angular velocity (about the rotor-furl axis), (deg/sec) Rotor-furl angular acceleration (about the rotor-furl axis), (deg/sec²)

Yaw Motions:

 $NacYaw = NacYawP = YawPzn = YawPzp = \left(\frac{180}{\pi}\right)q_{Yaw}$ $NacYawV = YawVzn = YawVzp = \left(\frac{180}{\pi}\right)\dot{q}_{Yaw}$ $NacYawA = YawAzn = YawAzp = \left(\frac{180}{\pi}\right)\ddot{q}_{Yaw}$ NacYawErr = HorWndDir - NacYaw - YawBrRDzt - PtfmYaw

Tower-Top Motions: $YawBrTDxp = \left\lceil r^{zo} - (TowerHt + Ptfm Re f)a_2 \right\rceil \cdot b_1$ (directed along the xp-axis), (m) $YawBrTDyp = -\left[\mathbf{r}^{\mathbf{z}\mathbf{o}} - (TowerHt + Ptfm \, Re \, f) \mathbf{a}_2 \right] \cdot \mathbf{b}_3$ (directed along the yp-axis), (m) $YawBrTDzp = \left[\mathbf{r}^{\mathbf{z}\mathbf{o}} - (TowerHt + Ptfm \, Re \, f) \mathbf{a}_2 \right] \cdot \mathbf{b}_2$ (directed along the zp-axis), (m) $TTDspFA = YawBrTDxt = \left[r^{zo} - (TowerHt + Ptfm Re f) a_2 \right] \cdot a_1$ position) (directed along the xt-axis), (m) $TTDspSS = YawBrTDyt = -\left[\mathbf{r}^{\mathbf{z}\mathbf{o}} - \left(TowerHt + Ptfm \, Re \, f \right) \mathbf{a}_2 \right] \cdot \mathbf{a}_3$ undeflected position) (directed along the yt-axis), (m) $TTDspAx = YawBrTDzt = \left[r^{zo} - (TowerHt + Ptfm Re f)a_2 \right] \cdot a_2$ position) (directed along the zt-axis), (m) $YawBrTAxp = {}^{E}a^{O} \cdot b_{I}$ $YawBrTAyp = -{}^{E}a^{O} \cdot b_{3}$ $YawBrTAzp = {}^{E}a^{O} \cdot b_{2}$ $TTDspRoll = YawBrRDxt = \left(\frac{180}{\pi}\right)^{X} \boldsymbol{\theta}^{B} \cdot \boldsymbol{a}_{1}$ (deg)

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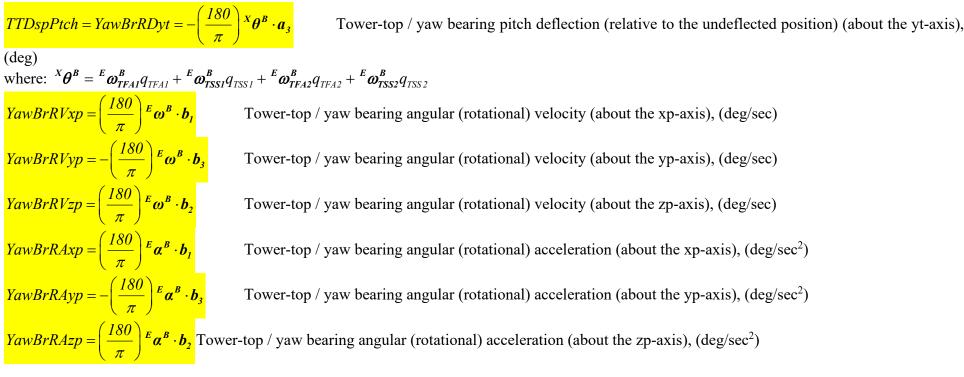
Nacelle yaw angle (position) (about the zn-/zp-axis), (deg)

Nacelle yaw angular velocity (about the zn-/zp-axis), (deg/sec)

Nacelle yaw angular acceleration (about the zn-/zp-axis), (deg/sec²) Nacelle yaw error (about the zt-axis), (deg)

Ht + Ptfm Re f)a_] · b_1Tower-top / yaw bearing translational deflection (relative to undeflected position)(m)Tower-top / yaw bearing translational deflection (relative to undeflected position)(m)Tower-top / yaw bearing translational deflection (relative to undeflected position)(m)Tower-top / yaw bearing translational deflection (relative to undeflected position)(m)Tower-top / yaw bearing translational deflection (relative to undeflected position)(m)Tower-top / yaw bearing translational deflection (relative to undeflected position)(m)Tower-top / yaw bearing fore-aft (translational) deflection (relative to undeflected position)(m)Tower-top / yaw bearing side-to-side (translational) deflection (relative to undeflected position)(m)Tower-top / yaw bearing side-to-side (translational) deflection (relative to undeflected position)(m)Tower-top / yaw bearing axial (translational) deflection (relative to undeflected position)(m)Tower-top / yaw bearing axial (translational) deflection (relative to undeflected position)(m)Tower-top / yaw bearing axial (translational) deflection (relative to undeflected position)(m)Tower-top / yaw bearing translational acceleration (directed along the xp-axis), (m/sec²)Tower-top / yaw bearing translational acceleration (directed along the xp-axis), (m/sec²)Tower-top / yaw bearing translational acceleration (directed along the zp-axis), (m/sec²)

Tower-top / yaw bearing roll deflection (relative to the undeflected position) (about the xt-axis),



 $\underline{\text{Tower Local Gage Motions:}} \\
 \underline{\text{TwHtiALxt}} = {}^{E} a^{T} \left(H^{\text{Node }i} \right) \cdot t_{I} \left(H^{\text{Node }i} \right) \\
 1,2,...,5), (\text{m/sec}^{2}) \\
 \underline{\text{TwHtiALyt}} = -{}^{E} a^{T} \left(H^{\text{Node }i} \right) \cdot t_{3} \left(H^{\text{Node }i} \right) \\
 (i = 1,2,...,5), (\text{m/sec}^{2}) \\
 \underline{\text{TwHtiALzt}} = {}^{E} a^{T} \left(H^{\text{Node }i} \right) \cdot t_{2} \left(H^{\text{Node }i} \right) \\
 1,2,...,5), (\text{m/sec}^{2})$

Tower local fore-aft translational acceleration (absolute) of node *i* (directed along the *local* xt-axis) (i =Tower local side-to-side translational acceleration (absolute) of node *i* (directed along the *local* yt-axis) Tower local axial translational acceleration (absolute) of node *i* (directed along the *local* zt-axis) (i =

<u>Tail-Furl Motions:</u> $TailFurl = TailFurlP = \left(\frac{180}{\pi}\right)q_{TFrl}$ $TailFurlV = \left(\frac{180}{\pi}\right)\dot{q}_{TFrl}$ $TailFurlA = \left(\frac{180}{\pi}\right)\ddot{q}_{TFrl}$

Platform Motions: $PtfmTDxt = \mathbf{r}^{\mathbf{Z}} \cdot \mathbf{a}_{\mathbf{I}}$ $PtfmTDyt = -\mathbf{r}^{\mathbf{Z}} \cdot \mathbf{a}_{3}$ $PtfmTDzt = \mathbf{r}^{\mathbf{Z}} \cdot \mathbf{a}_{2}$ $PtfmSurge = PtfmTDxi = q_{sa}$ $PtfmSway = PtfmTDyi = q_{Sw}$ $PtfmHeave = PtfmTDzi = q_{Hv}$ $PtfmTVxt = {}^{E}v^{Z} \cdot a_{I}$ $PtfmTVyt = -{}^{E}v^{Z} \cdot a_{3}$ $PtfmTVzt = {}^{E}v^{Z} \cdot a,$ $PtfmTVxi = \dot{q}_{sa}$ $PtfmTVyi = \dot{q}_{su}$ $PtfmTVzi = \dot{q}_{Hy}$ $PtfmTAxt = {^{E}a^{Z}} \cdot a_{1}$ $PtfmTAyt = -{}^{E}a^{Z} \cdot a_{3}$ $PtfmTAzt = {}^{E}a^{Z} \cdot a,$ $PtfmTAxi = \ddot{q}_{s_{\alpha}}$ $PtfmTAyi = \ddot{q}_{Sw}$ $PtfmTAzi = \ddot{q}_{Hv}$

Tail-furl angle (position) (about the tail-furl axis), (deg)

Tail-furl angular velocity (about the tail-furl axis), (deg/sec)

Tail-furl angular acceleration (about the tail-furl axis), (deg/sec²)

Platform horizontal surge displacement (directed along the xt-axis), (m) Platform horizontal sway displacement (directed along the yt-axis), (m) Platform vertical heave displacement (directed along the zt-axis), (m) Platform horizontal surge displacement (directed along the xi-axis), (m) Platform horizontal sway displacement (directed along the yi-axis), (m) Platform vertical heave displacement (directed along the zi-axis), (m) Platform horizontal surge velocity (directed along the xt-axis), (m/sec) Platform horizontal sway velocity (directed along the yt-axis), (m/sec) Platform vertical heave velocity (directed along the zt-axis), (m/sec) Platform horizontal surge velocity (directed along the xi-axis), (m/sec) Platform horizontal sway velocity (directed along the yi-axis), (m/sec) Platform vertical heave velocity (directed along the zi-axis), (m/sec) Platform horizontal surge acceleration (directed along the xt-axis), (m/sec²) Platform horizontal sway acceleration (directed along the yt-axis), (m/sec²) Platform vertical heave acceleration (directed along the zt-axis), (m/sec²) Platform horizontal surge acceleration (directed along the xi-axis), (m/sec²) Platform horizontal sway acceleration (directed along the yi-axis), (m/sec²) Platform vertical heave acceleration (directed along the zi-axis), (m/sec²)

 $PtfmRoll = PtfmRDxi = \left(\frac{180}{\pi}\right)q_R$ $PtfmPitch = PtfmRDyi = \left(\frac{180}{\pi}\right)q_{P}$ $PtfmYaw = PtfmRDzi = \left(\frac{180}{\pi}\right)q_{Y}$ $PtfmRVxt = \left(\frac{180}{\pi}\right)^{E} \boldsymbol{\omega}^{X} \cdot \boldsymbol{a}_{1}$ $PtfmRVyt = -\left(\frac{180}{\pi}\right)^{E} \boldsymbol{\omega}^{X} \cdot \boldsymbol{a}_{3}$ $PtfmRVzt = \left(\frac{180}{\pi}\right)^{E} \boldsymbol{\omega}^{X} \cdot \boldsymbol{a}_{2}$ $PtfmRVxi = \left(\frac{180}{\pi}\right)\dot{q}_R$ $PtfmRVyi = \left(\frac{180}{\pi}\right)\dot{q}_P$ $PtfmRVzi = \left(\frac{180}{\pi}\right)\dot{q}_{Y}$ $PtfmRAxt = \left(\frac{180}{\pi}\right)^{E} \boldsymbol{\alpha}^{X} \cdot \boldsymbol{a}_{I}$ $PtfmRAyt = -\left(\frac{180}{\pi}\right)^{E} \boldsymbol{\alpha}^{X} \cdot \boldsymbol{a}_{3}$ $PtfmRAzt = \left(\frac{180}{\pi}\right)^{E} \boldsymbol{\alpha}^{X} \cdot \boldsymbol{a}_{2}$ $PtfmRAxi = \left(\frac{180}{\pi}\right)\ddot{q}_R$ $PtfmRAyi = \left(\frac{180}{\pi}\right)\ddot{q}_{P}$

Platform roll tilt displacement (about the xi-axis), (deg) Platform pitch tilt displacement (about the yi-axis), (deg) Platform yaw displacement (about the zi-axis), (deg) Platform roll tilt velocity (about the xt-axis), (deg/sec) Platform pitch tilt velocity (about the yt-axis), (deg/sec) Platform yaw velocity (about the zt-axis), (deg/sec) Platform roll tilt velocity (about the xi-axis), (deg/sec) Platform pitch tilt velocity (about the yi-axis), (deg/sec) Platform yaw velocity (about the zi-axis), (deg/sec) Platform roll tilt acceleration (about the xt-axis), (deg/sec^2) Platform pitch tilt acceleration (about the yt-axis), (deg/sec^2) Platform yaw acceleration (about the zt-axis), (deg/sec²) Platform roll tilt acceleration (about the xi-axis), (deg/sec^2) Platform pitch tilt acceleration (about the yi-axis), (deg/sec^2) Platform yaw acceleration (about the zi-axis), (deg/sec²)

Tail-Furl Motions:				
$TFinAlpha = \left(\frac{180}{\pi}\right)TFinAOA$	Т			
TFinCLift = TFinCL	Т			
TFinCDrag = TFinCD	Т			
TFinDn Pr s = TFinQ	Т			
TFinCPFx = TFinKFx / 1,000	Т			
TFinCPFy = TFinKFy / 1,000	Т			

Tail fin angle of attack, (deg)

Tail fin lift coefficient, (-) Tail fin drag coefficient, (-) Tail fin dynamic pressure, (Pa) Tail fin tangential force, (kN) Tail fin normal force, (kN)

Wind Motions:

Wind Wittins.		
WindVxi = uWind	<i>Vind</i> Nominal hub-height wind velocity (directed along the xi-axis), (m/s)	
<i>WindVyi</i> = <i>vWind</i>	dVyi = vWind Cross-wind hub-height velocity (directed along the yi-axis), (m/s)	
<i>WindVzi</i> = <i>wWind</i> Vertical hub-height wind velocity (directed along the zi-axis), (m/s)		
$TotWindV = \sqrt{Win}$	dVxi ² + WindVyi ² + WindVzi ²	Total hub-height wind velocity magnitude, (m/s)
$HorWindV = \sqrt{Win}$	ndVxi ² +WindVyi ²	Horizontal hub-height wind velocity magnitude (in the xi-/yi-plane), (m/s)
<i>HorWndDir</i> Horizontal hub-height wind direction (about the zi-axis), (deg)		
VerWndDir	Vertical hub-height wind direction (about an axis orthogonal to the zi-axis and the horizontal wind vector), (deg)	

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