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There are several points on a 2-bladed turbine: Z (platform reference), Y (platform mass center), T (tower node), O (tower-top / base-plate / yaw bearing mass center), U (nacelle mass center), V (arbitrary point on rotor-furl axis), W (arbitrary point on tail-furl axis), D (center of mass of structure that furls with the rotor [not including rotor]), IMU (nacelle inertial measurement unit), P (teeter pin), SG [shaft strain gage location: i.e., a point on the shaft a distance *ShftGagL* towards the nacelle from point P (or point Q for a 3-blader since point P does not exist)], Q (apex of coning angle), C (hub mass center), S1 (blade node for blade 1), S2 (blade node for blade 2), I (tail boom mass center), J (tail fin mass center), and K (tail fin center-of-pressure). There are also several reference frames: E (earth / inertial), X (platform / tower base), F (tower element body), B (tower-top / base plate), N (nacelle), R (structure that furls with the rotor—generator housing, etc…), L (low speed shaft on rotor end of LSS-compliance), H (hub / rotor), M1 (blade 1 element body), M2 (blade 2 element body), G (fixed in the high speed shaft / generator), and A (tail). The following are derivations of the position vectors, angular velocities, linear velocities, partial angular velocities, partial linear velocities, angular accelerations, and linear accelerations of all these points on the 2-bladed turbine (point SG's velocities and accelerations are not derived since they wont be used in the ensuing analysis). The velocities and accelerations of points on a 3-bladed turbine are very similar.

Position Vectors:

$$\boldsymbol{r}^{\boldsymbol{Z}} = q_{Sg}\boldsymbol{z_1} + q_{Hv}\boldsymbol{z_2} - q_{Sw}\boldsymbol{z_3}$$ (relative to the ground)

$$\boldsymbol{r}^{\boldsymbol{ZY}} = \left(PtfmRef - PtfmCM \right)\boldsymbol{a_2}$$

$$\begin{aligned}\boldsymbol{r}^{\boldsymbol{ZT}}(h) = {} & \left[\phi_1^{TFA}(h) q_{TFA1} + \phi_2^{TFA}(h) q_{TFA2} \right]\boldsymbol{a_1} \\ & + \left\{ h + PtfmRef - TwrDraft + TwrRBHt - \frac{1}{2}\left[\begin{aligned} & S_{11}^{TFA}(h) q_{TFA1}^2 + S_{22}^{TFA}(h) q_{TFA2}^2 + 2S_{12}^{TFA}(h) q_{TFA1} q_{TFA2} \\ & + S_{11}^{TSS}(h) q_{TSS1}^2 + S_{22}^{TSS}(h) q_{TSS2}^2 + 2S_{12}^{TSS}(h) q_{TSS1} q_{TSS2} \end{aligned} \right] \right\}\boldsymbol{a_2} \\ & + \left[\phi_1^{TSS}(h) q_{TSS1} + \phi_2^{TSS}(h) q_{TSS2} \right]\boldsymbol{a_3}\end{aligned}$$

where,

$$S_{ij}^{TFA}(h) = \int_0^h \left[\frac{d\phi_i^{TFA}(h')}{dh'} \frac{d\phi_j^{TFA}(h')}{dh'} \right] dh' \quad (i, j = 1, 2)$$ (which is symmetric)

and

$$S_{ij}^{TSS}(h) = \int_0^h \left[\frac{d\phi_i^{TSS}(h')}{dh'} \frac{d\phi_j^{TSS}(h')}{dh'} \right] dh' \quad (i, j = 1, 2)$$ (which is symmetric)

Note limits on h: $0 \le h \le TowerHt + TwrDraft - TwrRBHt = TwrFlexL$

$$\boldsymbol{r}^{\boldsymbol{ZO}} = \left[q_{TFA1} + q_{TFA2} \right] \boldsymbol{a_1} + \left\{ PtfmRef + TowerHt - \frac{1}{2} \left[\begin{array}{l} S_{11}^{TFA}\left(TwrFlexL\right) q_{TFA1}^2 + S_{22}^{TFA}\left(TwrFlexL\right) q_{TFA2}^2 + 2S_{12}^{TFA}\left(TwrFlexL\right) q_{TFA1} q_{TFA2} \\ + S_{11}^{TSS}\left(TwrFlexL\right) q_{TSS1}^2 + S_{22}^{TSS}\left(TwrFlexL\right) q_{TSS2}^2 + 2S_{12}^{TSS}\left(TwrFlexL\right) q_{TSS1} q_{TSS2} \end{array} \right] \right\} \boldsymbol{a_2} + \left[q_{TSS1} + q_{TSS2} \right] \boldsymbol{a_3}$$

$$\boldsymbol{r}^{\boldsymbol{OU}} = NacCMxn\boldsymbol{d_1} + NacCMzn\boldsymbol{d_2} - NacCMyn\boldsymbol{d_3}$$

$$\boldsymbol{r}^{\boldsymbol{OV}} = RFrlPntxn\boldsymbol{d_1} + RFrlPntzn\boldsymbol{d_2} - RFrlPntyn\boldsymbol{d_3}$$

$$\boldsymbol{r}^{\boldsymbol{VD}} = \left(RFrlCMxn - RFrlPntxn\right)\boldsymbol{rf_1} + \left(RFrlCMzn - RFrlPntzn\right)\boldsymbol{rf_2} - \left(RFrlCMyn - RFrlPntyn\right)\boldsymbol{rf_3}$$

$$\boldsymbol{r}^{\boldsymbol{VIMU}} = \left(NcIMUxn - RFrlPntxn\right)\boldsymbol{rf_1} + \left(NcIMUzn - RFrlPntzn\right)\boldsymbol{rf_2} - \left(NcIMUyn - RFrlPntyn\right)\boldsymbol{rf_3}$$

$$\boldsymbol{r}^{\boldsymbol{VP}} = -RFrlPntxn\boldsymbol{rf_1} + \left(Twr2Shft - RFrlPntzn\right)\boldsymbol{rf_2} - \left(Yaw2Shft - RFrlPntyn\right)\boldsymbol{rf_3} + OverHang\boldsymbol{c_1}$$

$$\boldsymbol{r}^{\boldsymbol{PSG}} = ShftGagL\boldsymbol{c_1}$$

$$\boldsymbol{r}^{\boldsymbol{PQ}} = -UndSling\boldsymbol{g_1}$$

$$\boldsymbol{r}^{\boldsymbol{QC}} = HubCM\boldsymbol{g_1}$$

$$\boldsymbol{r}^{\boldsymbol{QS1}}(r) = \left[\phi_1^{B1}(r) q_{B1F1} + \phi_2^{B1}(r) q_{B1F2} + \phi_3^{B1}(r) q_{B1E1} \right] \boldsymbol{j_1^{B1}} + \left[\psi_1^{B1}(r) q_{B1F1} + \psi_2^{B1}(r) q_{B1F2} + \psi_3^{B1}(r) q_{B1E1} \right] \boldsymbol{j_2^{B1}} + \left\{ r + HubRad - \frac{1}{2} \left[S_{11}^{B1}(r) q_{B1F1}^2 + S_{22}^{B1}(r) q_{B1F2}^2 + S_{33}^{B1}(r) q_{B1E1}^2 + 2S_{12}^{B1}(r) q_{B1F1} q_{B1F2} + 2S_{23}^{B1}(r) q_{B1F2} q_{B1E1} + 2S_{13}^{B1}(r) q_{B1F1} q_{B1E1} \right] \right\} \boldsymbol{j_3^{B1}}$$

where,

$$S_{ij}^{B1}(r)=\int_0^r\left[\frac{d\phi_i^{B1}(r')}{dr'}\frac{d\phi_j^{B1}(r')}{dr'}+\frac{d\psi_i^{B1}(r')}{dr'}\frac{d\psi_j^{B1}(r')}{dr'}\right]dr' \quad (i,j=1,2,3)$$ (which is symmetric)

and the twisted shape functions are defined as follows:

$$\phi_1^{B1}(r)=\int_0^r\left\{\int_0^{r'}\frac{d^2\phi_1^{B1F}(r'')}{dr''^2}cos\left[\theta_S^{B1}(r'')\right]dr''\right\}dr',$$

$$\psi_1^{B1}(r)=-\int_0^r\left\{\int_0^{r'}\frac{d^2\phi_1^{B1F}(r'')}{dr''^2}sin\left[\theta_S^{B1}(r'')\right]dr''\right\}dr',$$

$$\phi_2^{B1}(r)=\int_0^r\left\{\int_0^{r'}\frac{d^2\phi_2^{B1F}(r'')}{dr''^2}cos\left[\theta_S^{B1}(r'')\right]dr''\right\}dr',$$

$$\psi_2^{B1}(r)=-\int_0^r\left\{\int_0^{r'}\frac{d^2\phi_2^{B1F}(r'')}{dr''^2}sin\left[\theta_S^{B1}(r'')\right]dr''\right\}dr',$$

$$\phi_3^{B1}(r)=\int_0^r\left\{\int_0^{r'}\frac{d^2\phi_1^{B1E}(r'')}{dr''^2}sin\left[\theta_S^{B1}(r'')\right]dr''\right\}dr',$$ and

$$\psi_3^{B1}(r)=\int_0^r\left\{\int_0^{r'}\frac{d^2\phi_1^{B1E}(r'')}{dr''^2}cos\left[\theta_S^{B1}(r'')\right]dr''\right\}dr'$$

The equation for $\boldsymbol{r}^{QS2}(r)$ is similar.

Note limit on r: $0\le r\le TipRad-HubRad=BldFlexL$

$$\boldsymbol{r}^{OW}=TFrlPntxn\boldsymbol{d}_1+TFrlPntzn\boldsymbol{d}_2-TFrlPntyn\boldsymbol{d}_3$$

$$\boldsymbol{r}^{WI}=(BoomCMxn-TFrlPntxn)\boldsymbol{tf}_1+(BoomCMzn-TFrlPntzn)\boldsymbol{tf}_2-(BoomCMyn-TFrlPntyn)\boldsymbol{tf}_3$$

$$\boldsymbol{r}^{WJ}=(TFinCMxn-TFrlPntxn)\boldsymbol{tf}_1+(TFinCMzn-TFrlPntzn)\boldsymbol{tf}_2-(TFinCMyn-TFrlPntyn)\boldsymbol{tf}_3$$

$$\boldsymbol{r}^{WK}=(TFinCPxn-TFrlPntxn)\boldsymbol{tf}_1+(TFinCPzn-TFrlPntzn)\boldsymbol{tf}_2-(TFinCPyn-TFrlPntyn)\boldsymbol{tf}_3$$

Angular Velocities:

$${}^{E}\boldsymbol{\omega}^{X} = \dot{q}_R \boldsymbol{z_1} + \dot{q}_Y \boldsymbol{z_2} - \dot{q}_P \boldsymbol{z_3}$$

$${}^{E}\boldsymbol{\omega}^{F}(h) = {}^{E}\boldsymbol{\omega}^{X} + \left[\frac{d\phi_1^{TSS}(h)}{dh}\dot{q}_{TSS1} + \frac{d\phi_2^{TSS}(h)}{dh}\dot{q}_{TSS2}\right]\boldsymbol{a_1} - \left[\frac{d\phi_1^{TFA}(h)}{dh}\dot{q}_{TFA1} + \frac{d\phi_2^{TFA}(h)}{dh}\dot{q}_{TFA2}\right]\boldsymbol{a_3}$$

$${}^{E}\boldsymbol{\omega}^{B} = {}^{E}\boldsymbol{\omega}^{X} + \left[\left.\frac{d\phi_1^{TSS}(h)}{dh}\right|_{h=TwrFlexL}\dot{q}_{TSS1} + \left.\frac{d\phi_2^{TSS}(h)}{dh}\right|_{h=TwrFlexL}\dot{q}_{TSS2}\right]\boldsymbol{a_1} - \left[\left.\frac{d\phi_1^{TFA}(h)}{dh}\right|_{h=TwrFlexL}\dot{q}_{TFA1} + \left.\frac{d\phi_2^{TFA}(h)}{dh}\right|_{h=TwrFlexL}\dot{q}_{TFA2}\right]\boldsymbol{a_3}$$

$${}^{E}\boldsymbol{\omega}^{N} = {}^{E}\boldsymbol{\omega}^{B} + \dot{q}_{Yaw}\boldsymbol{d_2}$$

$${}^{E}\boldsymbol{\omega}^{R} = {}^{E}\boldsymbol{\omega}^{N} + \dot{q}_{RFrl}\boldsymbol{rfa}$$

where, $\boldsymbol{rfa} = cos(RFrlSkew)cos(RFrlTilt)\boldsymbol{d_1} + sin(RFrlTilt)\boldsymbol{d_2} - sin(RFrlSkew)cos(RFrlTilt)\boldsymbol{d_3}$

$${}^{E}\boldsymbol{\omega}^{L} = {}^{E}\boldsymbol{\omega}^{R} + \dot{q}_{DrTr}\boldsymbol{c_1} + \dot{q}_{GeAz}\boldsymbol{c_1}$$

$${}^{E}\boldsymbol{\omega}^{H} = {}^{E}\boldsymbol{\omega}^{L} + \dot{q}_{Teet}\boldsymbol{f_2}$$

$${}^{E}\boldsymbol{\omega}^{M1}(r) = {}^{E}\boldsymbol{\omega}^{H} - \left[\frac{d\psi_1^{B1}(r)}{dr}\dot{q}_{B1F1} + \frac{d\psi_2^{B1}(r)}{dr}\dot{q}_{B1F2} + \frac{d\psi_3^{B1}(r)}{dr}\dot{q}_{B1E1}\right]\boldsymbol{j_1^{B1}} + \left[\frac{d\phi_1^{B1}(r)}{dr}\dot{q}_{B1F1} + \frac{d\phi_2^{B1}(r)}{dr}\dot{q}_{B1F2} + \frac{d\phi_3^{B1}(r)}{dr}\dot{q}_{B1E1}\right]\boldsymbol{j_2^{B1}}$$

The equation for ${}^{E}\boldsymbol{\omega}^{M2}(r)$ is similar.

Since the generator is attached to the high speed shaft which may or may not rotate in the opposite direction of the low speed shaft and since q_{GeAz} represents the position of the low speed shaft near the entrance of the gearbox,

$${}^{E}\boldsymbol{\omega}^{G} = {}^{E}\boldsymbol{\omega}^{R} + GenDir \cdot GBRatio \cdot \dot{q}_{GeAz}\boldsymbol{c_1}$$

where,

$$GenDir = \begin{cases} -1 & for \quad GB\,Reverse = True \\ 1 & for \quad GB\,Reverse = False \end{cases}$$

$$^{E}\boldsymbol{\omega}^{A} = {}^{E}\boldsymbol{\omega}^{N} + \dot{q}_{TFrl}\boldsymbol{tfa}$$

where, $\boldsymbol{tfa} = cos\left(TFrlSkew\right)cos\left(TFrlTilt\right)\boldsymbol{d_1} + sin\left(TFrlTilt\right)\boldsymbol{d_2} - sin\left(TFrlSkew\right)cos\left(TFrlTilt\right)\boldsymbol{d_3}$

Linear Velocities:

$$ {}^{E}\boldsymbol{v}^{Z} = \dot{q}_{Sg}\boldsymbol{z_1} + \dot{q}_{Hv}\boldsymbol{z_2} - \dot{q}_{Sw}\boldsymbol{z_3} $$

$$ {}^{E}\boldsymbol{v}^{Y} = {}^{E}\boldsymbol{v}^{Z} + {}^{E}\boldsymbol{\omega}^{X} \times \boldsymbol{r}^{ZY} $$

$$ {}^{E}\boldsymbol{v}^{T}(h) = {}^{E}\boldsymbol{v}^{Z} + {}^{X}\boldsymbol{v}^{T}(h) + {}^{E}\boldsymbol{\omega}^{X} \times \boldsymbol{r}^{ZT}(h) $$

where,

$$ \begin{aligned} {}^{X}\boldsymbol{v}^{T}(h) = &\left[\phi_1^{TFA}(h)\dot{q}_{TFA1} + \phi_2^{TFA}(h)\dot{q}_{TFA2}\right]\boldsymbol{a_1} \\ &-\left[\begin{aligned} &S_{11}^{TFA}(h)q_{TFA1}\dot{q}_{TFA1} + S_{22}^{TFA}(h)q_{TFA2}\dot{q}_{TFA2} + S_{12}^{TFA}(h)\left(\dot{q}_{TFA1}q_{TFA2} + q_{TFA1}\dot{q}_{TFA2}\right) \\ &+ S_{11}^{TSS}(h)q_{TSS1}\dot{q}_{TSS1} + S_{22}^{TSS}(h)q_{TSS2}\dot{q}_{TSS2} + S_{12}^{TSS}(h)\left(\dot{q}_{TSS1}q_{TSS2} + q_{TSS1}\dot{q}_{TSS2}\right) \end{aligned}\right]\boldsymbol{a_2} \\ &+\left[\phi_1^{TSS}(h)\dot{q}_{TSS1} + \phi_2^{TSS}(h)\dot{q}_{TSS2}\right]\boldsymbol{a_3} \end{aligned} $$

$$ {}^{E}\boldsymbol{v}^{O} = {}^{E}\boldsymbol{v}^{Z} + {}^{X}\boldsymbol{v}^{O} + {}^{E}\boldsymbol{\omega}^{X} \times \boldsymbol{r}^{ZO} $$

where,

$$ \begin{aligned} {}^{X}\boldsymbol{v}^{O} = &\left[\dot{q}_{TFA1} + \dot{q}_{TFA2}\right]\boldsymbol{a_1} \\ &-\left[\begin{aligned} &S_{11}^{TFA}(TwrFlexL)q_{TFA1}\dot{q}_{TFA1} + S_{22}^{TFA}(TwrFlexL)q_{TFA2}\dot{q}_{TFA2} + S_{12}^{TFA}(TwrFlexL)\left(\dot{q}_{TFA1}q_{TFA2} + q_{TFA1}\dot{q}_{TFA2}\right) \\ &+ S_{11}^{TSS}(TwrFlexL)q_{TSS1}\dot{q}_{TSS1} + S_{22}^{TSS}(TwrFlexL)q_{TSS2}\dot{q}_{TSS2} + S_{12}^{TSS}(TwrFlexL)\left(\dot{q}_{TSS1}q_{TSS2} + q_{TSS1}\dot{q}_{TSS2}\right) \end{aligned}\right]\boldsymbol{a_2} \\ &+\left[\dot{q}_{TSS1} + \dot{q}_{TSS2}\right]\boldsymbol{a_3} \end{aligned} $$

$$ {}^{E}\boldsymbol{v}^{U} = {}^{E}\boldsymbol{v}^{O} + {}^{E}\boldsymbol{\omega}^{N} \times \boldsymbol{r}^{OU} $$

$$ {}^{E}\boldsymbol{v}^{V} = {}^{E}\boldsymbol{v}^{O} + {}^{E}\boldsymbol{\omega}^{N} \times \boldsymbol{r}^{OV} $$

$$ {}^{E}\boldsymbol{v}^{D} = {}^{E}\boldsymbol{v}^{V} + {}^{E}\boldsymbol{\omega}^{R} \times \boldsymbol{r}^{VD} $$

The equation for ${}^{E}\boldsymbol{v}^{IMU}$ is similar.

$$ {}^{E}\boldsymbol{v}^{P} = {}^{E}\boldsymbol{v}^{V} + {}^{E}\boldsymbol{\omega}^{R} \times \boldsymbol{r}^{VP} $$

$${}^{E}\boldsymbol{v}^{Q} = {}^{E}\boldsymbol{v}^{P} + {}^{E}\boldsymbol{\omega}^{H} \times \boldsymbol{r}^{PQ}$$

$${}^{E}\boldsymbol{v}^{C} = {}^{E}\boldsymbol{v}^{Q} + {}^{E}\boldsymbol{\omega}^{H} \times \boldsymbol{r}^{QC}$$

$${}^{E}\boldsymbol{v}^{S1}(r) = {}^{E}\boldsymbol{v}^{Q} + {}^{H}\boldsymbol{v}^{S1}(r) + {}^{E}\boldsymbol{\omega}^{H} \times \boldsymbol{r}^{QS1}(r)$$

where,

$$\begin{aligned} {}^{H}\boldsymbol{v}^{S1}(r) = & \left[\phi_{1}^{B1}(r)\dot{q}_{B1F1} + \phi_{2}^{B1}(r)\dot{q}_{B1F2} + \phi_{3}^{B1}(r)\dot{q}_{B1E1}\right]\boldsymbol{j}_{1}^{B1} + \left[\psi_{1}^{B1}(r)\dot{q}_{B1F1} + \psi_{2}^{B1}(r)\dot{q}_{B1F2} + \psi_{3}^{B1}(r)\dot{q}_{B1E1}\right]\boldsymbol{j}_{2}^{B1} \\ & - \left[\begin{aligned} & S_{11}^{B1}(r)q_{B1F1}\dot{q}_{B1F1} + S_{22}^{B1}(r)q_{B1F2}\dot{q}_{B1F2} + S_{33}^{B1}(r)q_{B1E1}\dot{q}_{B1E1} \\ & + S_{12}^{B1}(r)\left(\dot{q}_{B1F1}q_{B1F2} + q_{B1F1}\dot{q}_{B1F2}\right) + S_{23}^{B1}(r)\left(\dot{q}_{B1F2}q_{B1E1} + q_{B1F2}\dot{q}_{B1E1}\right) + S_{13}^{B1}(r)\left(\dot{q}_{B1F1}q_{B1E1} + q_{B1F1}\dot{q}_{B1E1}\right) \end{aligned}\right]\boldsymbol{j}_{3}^{B1} \end{aligned}$$

The equation for ${}^{E}\boldsymbol{v}^{S2}(r)$ is similar.

$${}^{E}\boldsymbol{v}^{W} = {}^{E}\boldsymbol{v}^{O} + {}^{E}\boldsymbol{\omega}^{N} \times \boldsymbol{r}^{OW}$$

$${}^{E}\boldsymbol{v}^{I} = {}^{E}\boldsymbol{v}^{W} + {}^{E}\boldsymbol{\omega}^{A} \times \boldsymbol{r}^{WI}$$

$${}^{E}\boldsymbol{v}^{J} = {}^{E}\boldsymbol{v}^{W} + {}^{E}\boldsymbol{\omega}^{A} \times \boldsymbol{r}^{WJ}$$

$${}^{E}\boldsymbol{v}^{K} = {}^{E}\boldsymbol{v}^{W} + {}^{E}\boldsymbol{\omega}^{A} \times \boldsymbol{r}^{WK}$$

Partial Angular Velocities:

Recall that: ${}^{E}\boldsymbol{\omega}^{N_i}\left(\dot{q},q,t\right)=\left(\sum_{r=1}^{22}{}^{E}\boldsymbol{\omega}_{r}^{N_i}\left(q,t\right)\dot{q}_r\right)+{}^{E}\boldsymbol{\omega}_{t}^{N_i}\left(q,t\right)$ for each rigid body N_i in the system. Note that all of the ${}^{E}\boldsymbol{\omega}_{t}^{N_i}$ terms are zero as will be shown.

$$
{}^{E}\boldsymbol{\omega}_{r}^{X}=\begin{cases}\boldsymbol{z}_1 & for \quad r=R\\ -\boldsymbol{z}_3 & for \quad r=P\\ \boldsymbol{z}_2 & for \quad r=Y\\ 0 & otherwise\end{cases}
$$

$$
{}^{E}\boldsymbol{\omega}_{t}^{X}=0
$$

$$
{}^{E}\boldsymbol{\omega}_{r}^{F}\left(h\right)={}^{E}\boldsymbol{\omega}_{r}^{X}+\begin{cases}-\dfrac{d\phi_1^{TFA}\left(h\right)}{dh}\boldsymbol{a}_3 & for \quad r=TFA1\\ \dfrac{d\phi_1^{TSS}\left(h\right)}{dh}\boldsymbol{a}_1 & for \quad r=TSS1\\ -\dfrac{d\phi_2^{TFA}\left(h\right)}{dh}\boldsymbol{a}_3 & for \quad r=TFA2\\ \dfrac{d\phi_2^{TSS}\left(h\right)}{dh}\boldsymbol{a}_1 & for \quad r=TSS2\\ 0 & otherwise\end{cases}
$$

$$
{}^{E}\boldsymbol{\omega}_{t}^{F}\left(h\right)=0
$$

$$
{}^{E}\boldsymbol{\omega}_r^{B} = {}^{E}\boldsymbol{\omega}_r^{X} + \begin{cases} -\left.\dfrac{d\phi_1^{TFA}(h)}{dh}\right|_{h=TwrFlexL} \boldsymbol{a}_3 & for \quad r = TFA1 \\ \left.\dfrac{d\phi_1^{TSS}(h)}{dh}\right|_{h=TwrFlexL} \boldsymbol{a}_1 & for \quad r = TSS1 \\ -\left.\dfrac{d\phi_2^{TFA}(h)}{dh}\right|_{h=TwrFlexL} \boldsymbol{a}_3 & for \quad r = TFA2 \\ \left.\dfrac{d\phi_2^{TSS}(h)}{dh}\right|_{h=TwrFlexL} \boldsymbol{a}_1 & for \quad r = TSS2 \\ 0 & otherwise \end{cases}
$$

$$
{}^{E}\boldsymbol{\omega}_t^{B} = 0
$$

$$
{}^{E}\boldsymbol{\omega}_r^{N} = {}^{E}\boldsymbol{\omega}_r^{B} + \begin{cases} \boldsymbol{d}_2 & for \quad r = Yaw \\ 0 & otherwise \end{cases}
$$

$$
{}^{E}\boldsymbol{\omega}_t^{N} = 0
$$

$$
{}^{E}\boldsymbol{\omega}_r^{R} = {}^{E}\boldsymbol{\omega}_r^{N} + \begin{cases} \boldsymbol{rfa} & for \quad r = RFrl \\ 0 & otherwise \end{cases}
$$

$$
{}^{E}\boldsymbol{\omega}_t^{R} = 0
$$

$$
{}^{E}\boldsymbol{\omega}_r^{L} = {}^{E}\boldsymbol{\omega}_r^{R} + \begin{cases} \boldsymbol{c}_1 & for \quad r = GeAz \\ \boldsymbol{c}_1 & for \quad r = DrTr \\ 0 & otherwise \end{cases}
$$

$$
{}^{E}\boldsymbol{\omega}_t^{L} = 0
$$

$$ {}^{E}\boldsymbol{\omega}_{r}^{H} = {}^{E}\boldsymbol{\omega}_{r}^{L} + \begin{cases} \boldsymbol{f}_{2} & for \quad r = Teet \\ 0 & otherwise \end{cases} $$

$$ {}^{E}\boldsymbol{\omega}_{t}^{H} = 0 $$

$$ {}^{E}\boldsymbol{\omega}_{r}^{M1}(r) = {}^{E}\boldsymbol{\omega}_{r}^{H} + \begin{cases} -\frac{d\psi_{1}^{B1}(r)}{dr}\boldsymbol{j}_{1}^{B1} + \frac{d\phi_{1}^{B1}(r)}{dr}\boldsymbol{j}_{2}^{B1} & for \quad r = B1F1 \\ -\frac{d\psi_{3}^{B1}(r)}{dr}\boldsymbol{j}_{1}^{B1} + \frac{d\phi_{3}^{B1}(r)}{dr}\boldsymbol{j}_{2}^{B1} & for \quad r = B1E1 \\ -\frac{d\psi_{2}^{B1}(r)}{dr}\boldsymbol{j}_{1}^{B1} + \frac{d\phi_{2}^{B1}(r)}{dr}\boldsymbol{j}_{2}^{B1} & for \quad r = B1F2 \\ 0 & otherwise \end{cases} $$

$$ {}^{E}\boldsymbol{\omega}_{t}^{M1}(r) = 0 $$

The equations for ${}^{E}\boldsymbol{\omega}_{r}^{M2}(r)$ and ${}^{E}\boldsymbol{\omega}_{t}^{M2}(r)$ are similar.

$$ {}^{E}\boldsymbol{\omega}_{r}^{G} = {}^{E}\boldsymbol{\omega}_{r}^{R} + \begin{cases} GenDir \cdot GBRatio\boldsymbol{c}_{1} & for \quad r = GeAz \\ 0 & otherwise \end{cases} $$

$$ {}^{E}\boldsymbol{\omega}_{t}^{G} = 0 $$

$$ {}^{E}\boldsymbol{\omega}_{r}^{A} = {}^{E}\boldsymbol{\omega}_{r}^{N} + \begin{cases} \boldsymbol{tfa} & for \quad r = TFrl \\ 0 & otherwise \end{cases} $$

$$ {}^{E}\boldsymbol{\omega}_{t}^{A} = 0 $$

Partial Linear Velocities:

Recall that: ${}^{E}\boldsymbol{v}^{X_i}\left(\dot{q},q,t\right)=\left(\sum_{r=1}^{22}{}^{E}\boldsymbol{v}_{r}^{X_i}\left(q,t\right)\dot{q}_r\right)+{}^{E}\boldsymbol{v}_{t}^{X_i}\left(q,t\right)$ for each point X_i in the system. Note that all of the ${}^{E}\boldsymbol{v}_{t}^{X_i}$ terms are zero as will be shown.

$$
{}^{E}\boldsymbol{v}_{r}^{Z}=\begin{cases}\boldsymbol{z}_1 & for \quad r=Sg\\ -\boldsymbol{z}_3 & for \quad r=Sw\\ \boldsymbol{z}_2 & for \quad r=Hv\\ 0 & otherwise\end{cases}
$$

$$
{}^{E}\boldsymbol{v}_{t}^{Z}=0
$$

$$
{}^{E}\boldsymbol{v}_{r}^{Y}={}^{E}\boldsymbol{v}_{r}^{Z}+\begin{cases}{}^{E}\boldsymbol{\omega}_{r}^{X}\times\boldsymbol{r}^{ZY} & for \quad r=4,5,6\\ 0 & otherwise\end{cases}
$$

$$
{}^{E}\boldsymbol{v}_{t}^{Y}=0
$$

$$
{}^{E}\boldsymbol{v}_{r}^{T}\left(h\right)={}^{E}\boldsymbol{v}_{r}^{Z}+\begin{cases}{}^{E}\boldsymbol{\omega}_{r}^{X}\times\boldsymbol{r}^{ZT}\left(h\right) & for \quad r=4,5,6\\ \phi_1^{TFA}\left(h\right)\boldsymbol{a}_1-\left[S_{11}^{TFA}\left(h\right)q_{TFA1}+S_{12}^{TFA}\left(h\right)q_{TFA2}\right]\boldsymbol{a}_2 & for \quad r=TFA1\\ \phi_1^{TSS}\left(h\right)\boldsymbol{a}_3-\left[S_{11}^{TSS}\left(h\right)q_{TSS1}+S_{12}^{TSS}\left(h\right)q_{TSS2}\right]\boldsymbol{a}_2 & for \quad r=TSS1\\ \phi_2^{TFA}\left(h\right)\boldsymbol{a}_1-\left[S_{22}^{TFA}\left(h\right)q_{TFA2}+S_{12}^{TFA}\left(h\right)q_{TFA1}\right]\boldsymbol{a}_2 & for \quad r=TFA2\\ \phi_2^{TSS}\left(h\right)\boldsymbol{a}_3-\left[S_{22}^{TSS}\left(h\right)q_{TSS2}+S_{12}^{TSS}\left(h\right)q_{TSS1}\right]\boldsymbol{a}_2 & for \quad r=TSS2\\ 0 & otherwise\end{cases}
$$

$$
{}^{E}\boldsymbol{v}_{t}^{T}\left(h\right)=0
$$

$$ {}^E\boldsymbol{v}_r^{O} = {}^E\boldsymbol{v}_r^{Z} + \begin{cases} {}^E\boldsymbol{\omega}_r^{X} \times \boldsymbol{r}^{ZO} & for \quad r = 4,5,6 \\ \boldsymbol{a_1} - \left[S_{11}^{TFA}\left(TwrFlexL\right) q_{TFA1} + S_{12}^{TFA}\left(TwrFlexL\right) q_{TFA2} \right] \boldsymbol{a_2} & for \quad r = TFA1 \\ \boldsymbol{a_3} - \left[S_{11}^{TSS}\left(TwrFlexL\right) q_{TSS1} + S_{12}^{TSS}\left(TwrFlexL\right) q_{TSS2} \right] \boldsymbol{a_2} & for \quad r = TSS1 \\ \boldsymbol{a_1} - \left[S_{22}^{TFA}\left(TwrFlexL\right) q_{TFA2} + S_{12}^{TFA}\left(TwrFlexL\right) q_{TFA1} \right] \boldsymbol{a_2} & for \quad r = TFA2 \\ \boldsymbol{a_3} - \left[S_{22}^{TSS}\left(TwrFlexL\right) q_{TSS2} + S_{12}^{TSS}\left(TwrFlexL\right) q_{TSS1} \right] \boldsymbol{a_2} & for \quad r = TSS2 \\ 0 & otherwise \end{cases} $$

$$ {}^E\boldsymbol{v}_t^{O} = 0 $$

$$ {}^E\boldsymbol{v}_r^{U} = {}^E\boldsymbol{v}_r^{O} + \begin{cases} {}^E\boldsymbol{\omega}_r^{N} \times \boldsymbol{r}^{OU} & for \quad r = 4,5,\ldots,11 \\ 0 & otherwise \end{cases} $$

$$ {}^E\boldsymbol{v}_t^{U} = 0 $$

$$ {}^E\boldsymbol{v}_r^{V} = {}^E\boldsymbol{v}_r^{O} + \begin{cases} {}^E\boldsymbol{\omega}_r^{N} \times \boldsymbol{r}^{OV} & for \quad r = 4,5,\ldots,11 \\ 0 & otherwise \end{cases} $$

$$ {}^E\boldsymbol{v}_t^{V} = 0 $$

$$ {}^E\boldsymbol{v}_r^{D} = {}^E\boldsymbol{v}_r^{V} + \begin{cases} {}^E\boldsymbol{\omega}_r^{R} \times \boldsymbol{r}^{VD} & for \quad r = 4,5,\ldots,12 \\ 0 & otherwise \end{cases} $$

$$ {}^E\boldsymbol{v}_t^{D} = 0 $$

The equations for ${}^E\boldsymbol{v}_r^{IMU}$ and ${}^E\boldsymbol{v}_t^{IMU}$ are similar.

$$ {}^E\boldsymbol{v}_r^{P} = {}^E\boldsymbol{v}_r^{V} + \begin{cases} {}^E\boldsymbol{\omega}_r^{R} \times \boldsymbol{r}^{VP} & for \quad r = 4,5,\ldots,12 \\ 0 & otherwise \end{cases} $$

$$ {}^E\boldsymbol{v}_t^{P} = 0 $$

$$ {}^{E}\boldsymbol{v}_r^{Q} = {}^{E}\boldsymbol{v}_r^{P} + \begin{cases} {}^{E}\boldsymbol{\omega}_r^{H} \times \boldsymbol{r}^{PQ} & for \quad r = 4,5,\ldots,14 \\ {}^{E}\boldsymbol{\omega}_{Teet}^{H} \times \boldsymbol{r}^{PQ} & for \quad r = Teet \\ 0 & otherwise \end{cases} $$

$$ {}^{E}\boldsymbol{v}_t^{Q} = 0 $$

$$ {}^{E}\boldsymbol{v}_r^{C} = {}^{E}\boldsymbol{v}_r^{Q} + \begin{cases} {}^{E}\boldsymbol{\omega}_r^{H} \times \boldsymbol{r}^{QC} & for \quad r = 4,5,\ldots,14 \\ {}^{E}\boldsymbol{\omega}_{Teet}^{H} \times \boldsymbol{r}^{QC} & for \quad r = Teet \\ 0 & otherwise \end{cases} $$

$$ {}^{E}\boldsymbol{v}_t^{C} = 0 $$

$$ {}^{E}\boldsymbol{v}_r^{S1}(r) = {}^{E}\boldsymbol{v}_r^{Q} + \begin{cases} {}^{E}\boldsymbol{\omega}_r^{H} \times \boldsymbol{r}^{QS1}(r) & for \quad r = 4,5,\ldots,14 \\ \phi_1^{B1}(r)\,\boldsymbol{j}_1^{B1} + \psi_1^{B1}(r)\,\boldsymbol{j}_2^{B1} - \left[S_{11}^{B1}(r)q_{B1F1} + S_{12}^{B1}(r)q_{B1F2} + S_{13}^{B1}(r)q_{B1E1}\right]\boldsymbol{j}_3^{B1} & for \quad r = B1F1 \\ \phi_3^{B1}(r)\,\boldsymbol{j}_1^{B1} + \psi_3^{B1}(r)\,\boldsymbol{j}_2^{B1} - \left[S_{33}^{B1}(r)q_{B1E1} + S_{23}^{B1}(r)q_{B1F2} + S_{13}^{B1}(r)q_{B1F1}\right]\boldsymbol{j}_3^{B1} & for \quad r = B1E1 \\ \phi_2^{B1}(r)\,\boldsymbol{j}_1^{B1} + \psi_2^{B1}(r)\,\boldsymbol{j}_2^{B1} - \left[S_{22}^{B1}(r)q_{B1F2} + S_{12}^{B1}(r)q_{B1F1} + S_{23}^{B1}(r)q_{B1E1}\right]\boldsymbol{j}_3^{B1} & for \quad r = B1F2 \\ {}^{E}\boldsymbol{\omega}_{Teet}^{H} \times \boldsymbol{r}^{QS1}(r) & for \quad r = Teet \\ 0 & otherwise \end{cases} $$

$$ {}^{E}\boldsymbol{v}_t^{S1}(r) = 0 $$

The equations for ${}^{E}\boldsymbol{v}_r^{S2}(r)$ and ${}^{E}\boldsymbol{v}_t^{S2}(r)$ are similar.

$$ {}^{E}\boldsymbol{v}_r^{W} = {}^{E}\boldsymbol{v}_r^{O} + \begin{cases} {}^{E}\boldsymbol{\omega}_r^{N} \times \boldsymbol{r}^{OW} & for \quad r = 4,5,\ldots,11 \\ 0 & otherwise \end{cases} $$

$$ {}^{E}\boldsymbol{v}_t^{W} = 0 $$

$$^{E}\boldsymbol{v}_{r}^{\boldsymbol{I}} = {}^{E}\boldsymbol{v}_{r}^{\boldsymbol{W}} + \begin{cases} {}^{E}\boldsymbol{\omega}_{r}^{A} \times \boldsymbol{r}^{\boldsymbol{WI}} & for \quad r = 4,5,\ldots,11 \\ {}^{E}\boldsymbol{\omega}_{TFrl}^{A} \times \boldsymbol{r}^{\boldsymbol{WI}} & for \quad r = TFrl \\ 0 & otherwise \end{cases}$$

$$^{E}\boldsymbol{v}_{t}^{\boldsymbol{I}} = 0$$

$$^{E}\boldsymbol{v}_{r}^{\boldsymbol{J}} = {}^{E}\boldsymbol{v}_{r}^{\boldsymbol{W}} + \begin{cases} {}^{E}\boldsymbol{\omega}_{r}^{A} \times \boldsymbol{r}^{\boldsymbol{WJ}} & for \quad r = 4,5,\ldots,11 \\ {}^{E}\boldsymbol{\omega}_{TFrl}^{A} \times \boldsymbol{r}^{\boldsymbol{WJ}} & for \quad r = TFrl \\ 0 & otherwise \end{cases}$$

$$^{E}\boldsymbol{v}_{t}^{\boldsymbol{J}} = 0$$

$$^{E}\boldsymbol{v}_{r}^{\boldsymbol{K}} = {}^{E}\boldsymbol{v}_{r}^{\boldsymbol{W}} + \begin{cases} {}^{E}\boldsymbol{\omega}_{r}^{A} \times \boldsymbol{r}^{\boldsymbol{WK}} & for \quad r = 4,5,\ldots,11 \\ {}^{E}\boldsymbol{\omega}_{TFrl}^{A} \times \boldsymbol{r}^{\boldsymbol{WK}} & for \quad r = TFrl \\ 0 & otherwise \end{cases}$$

$$^{E}\boldsymbol{v}_{t}^{\boldsymbol{K}} = 0$$

Angular Accelerations:

Recall that: ${}^{E}\boldsymbol{\alpha}^{N_i}\left(\ddot{q},\dot{q},q,t\right)=\left(\sum_{r=1}^{22}{}^{E}\boldsymbol{\omega}_r^{N_i}\left(q,t\right)\ddot{q}_r\right)+\left[\sum_{r=1}^{22}\frac{d}{dt}\left({}^{E}\boldsymbol{\omega}_r^{N_i}\left(q,t\right)\right)\dot{q}_r\right]+\frac{d}{dt}\left({}^{E}\boldsymbol{\omega}_t^{N_i}\left(q,t\right)\right)$ for each rigid body N_i in the system. Note that the $\frac{d}{dt}\left({}^{E}\boldsymbol{\omega}_r^{N_i}\right)$ terms are all vector functions of $\left(\dot{q},q,t\right)$ and that all of the $\frac{d}{dt}\left({}^{E}\boldsymbol{\omega}_t^{N_i}\right)$ terms are zero as will be shown.

$$\frac{d}{dt}\left({}^{E}\boldsymbol{\omega}_r^{X}\right)=0$$
$$\frac{d}{dt}\left({}^{E}\boldsymbol{\omega}_t^{X}\right)=0$$

$$\frac{d}{dt}\left[{}^{E}\boldsymbol{\omega}_r^{F}\left(h\right)\right]=\begin{cases}{}^{E}\boldsymbol{\omega}^{X}\times{}^{E}\boldsymbol{\omega}_r^{F}\left(h\right) & for\quad r=7,8,\ldots,10\\ 0 & otherwise\end{cases}$$
$$\frac{d}{dt}\left[{}^{E}\boldsymbol{\omega}_t^{F}\left(h\right)\right]=0$$

$$\frac{d}{dt}\left({}^{E}\boldsymbol{\omega}_r^{B}\right)=\begin{cases}{}^{E}\boldsymbol{\omega}^{X}\times{}^{E}\boldsymbol{\omega}_r^{B} & for\quad r=7,8,\ldots,10\\ 0 & otherwise\end{cases}$$
$$\frac{d}{dt}\left({}^{E}\boldsymbol{\omega}_t^{B}\right)=0$$

$$\frac{d}{dt}\left({}^{E}\boldsymbol{\omega}_r^{N}\right)=\frac{d}{dt}\left({}^{E}\boldsymbol{\omega}_r^{B}\right)+\begin{cases}{}^{E}\boldsymbol{\omega}^{B}\times{}^{E}\boldsymbol{\omega}_{Yaw}^{N} & for\quad r=Yaw\\ 0 & otherwise\end{cases}$$
$$\frac{d}{dt}\left({}^{E}\boldsymbol{\omega}_t^{N}\right)=0$$

$$\frac{d}{dt}\left({}^{E}\boldsymbol{\omega}_{r}^{R}\right)=\frac{d}{dt}\left({}^{E}\boldsymbol{\omega}_{r}^{N}\right)+\begin{cases}{}^{E}\boldsymbol{\omega}^{N}\times{}^{E}\boldsymbol{\omega}_{\boldsymbol{RFrl}}^{\boldsymbol{R}} & for\quad r=RFrl\\ 0 & otherwise\end{cases}$$

$$\frac{d}{dt}\left({}^{E}\boldsymbol{\omega}_{t}^{R}\right)=0$$

$$\frac{d}{dt}\left({}^{E}\boldsymbol{\omega}_{r}^{L}\right)=\frac{d}{dt}\left({}^{E}\boldsymbol{\omega}_{r}^{R}\right)+\begin{cases}{}^{E}\boldsymbol{\omega}^{R}\times{}^{E}\boldsymbol{\omega}_{\boldsymbol{GeAz}}^{\boldsymbol{L}} & for\quad r=GeAz\\ {}^{E}\boldsymbol{\omega}^{R}\times{}^{E}\boldsymbol{\omega}_{\boldsymbol{DrTr}}^{\boldsymbol{L}} & for\quad r=DrTr\\ 0 & otherwise\end{cases}$$

$$\frac{d}{dt}\left({}^{E}\boldsymbol{\omega}_{t}^{L}\right)=0$$

$$\frac{d}{dt}\left({}^{E}\boldsymbol{\omega}_{r}^{H}\right)=\frac{d}{dt}\left({}^{E}\boldsymbol{\omega}_{r}^{L}\right)+\begin{cases}{}^{E}\boldsymbol{\omega}^{H}\times{}^{E}\boldsymbol{\omega}_{\boldsymbol{Teet}}^{\boldsymbol{H}} & for\quad r=Teet\\ 0 & otherwise\end{cases}$$

$$\frac{d}{dt}\left({}^{E}\boldsymbol{\omega}_{t}^{H}\right)=0$$

$$\frac{d}{dt}\left[{}^{E}\boldsymbol{\omega}_{r}^{M1}\left(r\right)\right]=\frac{d}{dt}\left({}^{E}\boldsymbol{\omega}_{r}^{H}\right)+\begin{cases}{}^{E}\boldsymbol{\omega}^{H}\times{}^{E}\boldsymbol{\omega}_{\boldsymbol{B1F1}}^{\boldsymbol{M1}}\left(r\right) & for\quad r=B1F1\\ {}^{E}\boldsymbol{\omega}^{H}\times{}^{E}\boldsymbol{\omega}_{\boldsymbol{B1E1}}^{\boldsymbol{M1}}\left(r\right) & for\quad r=B1E1\\ {}^{E}\boldsymbol{\omega}^{H}\times{}^{E}\boldsymbol{\omega}_{\boldsymbol{B1F2}}^{\boldsymbol{M1}}\left(r\right) & for\quad r=B1F2\\ 0 & otherwise\end{cases}$$

$$\frac{d}{dt}\left[{}^{E}\boldsymbol{\omega}_{t}^{M1}\left(r\right)\right]=0$$

The equations for $\frac{d}{dt}\left[{}^{E}\boldsymbol{\omega}_{r}^{M2}\left(r\right)\right]$ and $\frac{d}{dt}\left[{}^{E}\boldsymbol{\omega}_{t}^{M2}\left(r\right)\right]$ are similar.

$$\frac{d}{dt}\left({}^{E}\boldsymbol{\omega}_{r}^{G}\right)=\frac{d}{dt}\left({}^{E}\boldsymbol{\omega}_{r}^{R}\right)+\begin{cases}{}^{E}\boldsymbol{\omega}^{R}\times{}^{E}\boldsymbol{\omega}_{GeAz}^{G} & for \quad r=GeAz\\ 0 & otherwise\end{cases}$$

$$\frac{d}{dt}\left({}^{E}\boldsymbol{\omega}_{t}^{G}\right)=0$$

$$\frac{d}{dt}\left({}^{E}\boldsymbol{\omega}_{r}^{A}\right)=\frac{d}{dt}\left({}^{E}\boldsymbol{\omega}_{r}^{N}\right)+\begin{cases}{}^{E}\boldsymbol{\omega}^{N}\times{}^{E}\boldsymbol{\omega}_{TFrl}^{A} & for \quad r=TFrl\\ 0 & otherwise\end{cases}$$

$$\frac{d}{dt}\left({}^{E}\boldsymbol{\omega}_{t}^{A}\right)=0$$

Linear Accelerations:

Recall that: ${}^{E}\boldsymbol{a}^{X_i}\left(\ddot{q},\dot{q},q,t\right)=\left(\sum_{r=1}^{22}{}^{E}\boldsymbol{v}_{r}^{X_i}\left(q,t\right)\ddot{q}_r\right)+\left[\sum_{r=1}^{22}\frac{d}{dt}\left({}^{E}\boldsymbol{v}_{r}^{X_i}\left(q,t\right)\right)\dot{q}_r\right]+\frac{d}{dt}\left({}^{E}\boldsymbol{v}_{t}^{X_i}\left(q,t\right)\right)$ for each point X_i in the system. Note that the $\frac{d}{dt}\left({}^{E}\boldsymbol{v}_{r}^{X_i}\right)$ terms are all vector functions of $\left(\dot{q},q,t\right)$ and that all of the $\frac{d}{dt}\left({}^{E}\boldsymbol{v}_{t}^{X_i}\right)$ terms are zero as will be shown.

$$\frac{d}{dt}\left({}^{E}\boldsymbol{v}_{r}^{Z}\right)=0$$
$$\frac{d}{dt}\left({}^{E}\boldsymbol{v}_{t}^{Z}\right)=0$$

$$\frac{d}{dt}\left({}^{E}\boldsymbol{v}_{r}^{Y}\right)=\begin{cases}{}^{E}\boldsymbol{\omega}_{r}^{X}\times\left({}^{E}\boldsymbol{\omega}^{X}\times\boldsymbol{r}^{ZY}\right) & for\quad r=4,5,6\\ 0 & otherwise\end{cases}$$
$$\frac{d}{dt}\left({}^{E}\boldsymbol{v}_{t}^{Y}\right)=0$$

$$\frac{d}{dt}\left[{}^{E}\boldsymbol{v}_{r}^{T}\left(h\right)\right]=\begin{cases}{}^{E}\boldsymbol{\omega}_{r}^{X}\times\left[{}^{X}\boldsymbol{v}^{T}\left(h\right)+{}^{E}\boldsymbol{\omega}^{X}\times\boldsymbol{r}^{ZT}\left(h\right)\right] & for\quad r=4,5,6\\ -\left[S_{11}^{TFA}\left(h\right)\dot{q}_{TFA1}+S_{12}^{TFA}\left(h\right)\dot{q}_{TFA1}\right]\boldsymbol{a}_2+{}^{E}\boldsymbol{\omega}^{X}\times{}^{E}\boldsymbol{v}_{TFA1}^{T}\left(h\right) & for\quad r=TFA1\\ -\left[S_{11}^{TSS}\left(h\right)\dot{q}_{TSS1}+S_{12}^{TSS}\left(h\right)\dot{q}_{TSS2}\right]\boldsymbol{a}_2+{}^{E}\boldsymbol{\omega}^{X}\times{}^{E}\boldsymbol{v}_{TSS1}^{T}\left(h\right) & for\quad r=TSS1\\ -\left[S_{22}^{TFA}\left(h\right)\dot{q}_{TFA2}+S_{12}^{TFA}\left(h\right)\dot{q}_{TFA1}\right]\boldsymbol{a}_2+{}^{E}\boldsymbol{\omega}^{X}\times{}^{E}\boldsymbol{v}_{TFA2}^{T}\left(h\right) & for\quad r=TFA2\\ -\left[S_{22}^{TSS}\left(h\right)\dot{q}_{TSS2}+S_{12}^{TSS}\left(h\right)\dot{q}_{TSS1}\right]\boldsymbol{a}_2+{}^{E}\boldsymbol{\omega}^{X}\times{}^{E}\boldsymbol{v}_{TSS2}^{T}\left(h\right) & for\quad r=TSS2\\ 0 & otherwise\end{cases}$$
$$\frac{d}{dt}\left[{}^{E}\boldsymbol{v}_{t}^{T}\left(h\right)\right]=0$$

$$\frac{d}{dt}\left({}^{E}\boldsymbol{v}_{r}^{\boldsymbol{O}}\right)=\begin{cases}{}^{E}\boldsymbol{\omega}_{r}^{X}\times\left({}^{X}\boldsymbol{v}^{\boldsymbol{O}}+{}^{E}\boldsymbol{\omega}^{X}\times\boldsymbol{r}^{\boldsymbol{ZO}}\right) & for\quad r=4,5,6\\ -\left[S_{11}^{TFA}\left(TwrFlexL\right)\dot{q}_{TFA1}+S_{12}^{TFA}\left(TwrFlexL\right)\dot{q}_{TFA1}\right]\boldsymbol{a}_{2}+{}^{E}\boldsymbol{\omega}^{X}\times{}^{E}\boldsymbol{v}_{\boldsymbol{TFA1}}^{\boldsymbol{O}} & for\quad r=TFA1\\ -\left[S_{11}^{TSS}\left(TwrFlexL\right)\dot{q}_{TSS1}+S_{12}^{TSS}\left(TwrFlexL\right)\dot{q}_{TSS2}\right]\boldsymbol{a}_{2}+{}^{E}\boldsymbol{\omega}^{X}\times{}^{E}\boldsymbol{v}_{\boldsymbol{TSS1}}^{\boldsymbol{O}} & for\quad r=TSS1\\ -\left[S_{22}^{TFA}\left(TwrFlexL\right)\dot{q}_{TFA2}+S_{12}^{TFA}\left(TwrFlexL\right)\dot{q}_{TFA1}\right]\boldsymbol{a}_{2}+{}^{E}\boldsymbol{\omega}^{X}\times{}^{E}\boldsymbol{v}_{\boldsymbol{TFA2}}^{\boldsymbol{O}} & for\quad r=TFA2\\ -\left[S_{22}^{TSS}\left(TwrFlexL\right)\dot{q}_{TSS2}+S_{12}^{TSS}\left(TwrFlexL\right)\dot{q}_{TSS1}\right]\boldsymbol{a}_{2}+{}^{E}\boldsymbol{\omega}^{X}\times{}^{E}\boldsymbol{v}_{\boldsymbol{TSS2}}^{\boldsymbol{O}} & for\quad r=TSS2\\ 0 & otherwise\end{cases}$$

$$\frac{d}{dt}\left({}^{E}\boldsymbol{v}_{t}^{\boldsymbol{O}}\right)=0$$

$$\frac{d}{dt}\left({}^{E}\boldsymbol{v}_{r}^{U}\right)=\frac{d}{dt}\left({}^{E}\boldsymbol{v}_{r}^{\boldsymbol{O}}\right)+\begin{cases}{}^{E}\boldsymbol{\omega}_{r}^{N}\times\left({}^{E}\boldsymbol{\omega}^{N}\times\boldsymbol{r}^{\boldsymbol{OU}}\right) & for\quad r=4,5,6\\ \frac{d}{dt}\left({}^{E}\boldsymbol{\omega}_{r}^{N}\right)\times\boldsymbol{r}^{\boldsymbol{OU}}+{}^{E}\boldsymbol{\omega}_{r}^{N}\times\left({}^{E}\boldsymbol{\omega}^{N}\times\boldsymbol{r}^{\boldsymbol{OU}}\right) & for\quad r=7,8,\ldots,11\\ 0 & otherwise\end{cases}$$

$$\frac{d}{dt}\left({}^{E}\boldsymbol{v}_{t}^{U}\right)=0$$

$$\frac{d}{dt}\left({}^{E}\boldsymbol{v}_{r}^{V}\right)=\frac{d}{dt}\left({}^{E}\boldsymbol{v}_{r}^{\boldsymbol{O}}\right)+\begin{cases}{}^{E}\boldsymbol{\omega}_{r}^{N}\times\left({}^{E}\boldsymbol{\omega}^{N}\times\boldsymbol{r}^{\boldsymbol{OV}}\right) & for\quad r=4,5,6\\ \frac{d}{dt}\left({}^{E}\boldsymbol{\omega}_{r}^{N}\right)\times\boldsymbol{r}^{\boldsymbol{OV}}+{}^{E}\boldsymbol{\omega}_{r}^{N}\times\left({}^{E}\boldsymbol{\omega}^{N}\times\boldsymbol{r}^{\boldsymbol{OV}}\right) & for\quad r=7,8,\ldots,11\\ 0 & otherwise\end{cases}$$

$$\frac{d}{dt}\left({}^{E}\boldsymbol{v}_{t}^{V}\right)=0$$

$$\frac{d}{dt}\left({}^{E}\boldsymbol{v}_{r}^{D}\right)=\frac{d}{dt}\left({}^{E}\boldsymbol{v}_{r}^{V}\right)+\begin{cases}{}^{E}\boldsymbol{\omega}_{r}^{R}\times\left({}^{E}\boldsymbol{\omega}^{R}\times\boldsymbol{r}^{VD}\right) & for \quad r=4,5,6\\ \frac{d}{dt}\left({}^{E}\boldsymbol{\omega}_{r}^{R}\right)\times\boldsymbol{r}^{VD}+{}^{E}\boldsymbol{\omega}_{r}^{R}\times\left({}^{E}\boldsymbol{\omega}^{R}\times\boldsymbol{r}^{VD}\right) & for \quad r=7,8,\ldots,12\\ 0 & otherwise\end{cases}$$

$$\frac{d}{dt}\left({}^{E}\boldsymbol{v}_{t}^{D}\right)=0$$

The equations for $\frac{d}{dt}\left({}^{E}\boldsymbol{v}_{r}^{IMU}\right)$ and $\frac{d}{dt}\left({}^{E}\boldsymbol{v}_{t}^{IMU}\right)$ are similar.

$$\frac{d}{dt}\left({}^{E}\boldsymbol{v}_{r}^{P}\right)=\frac{d}{dt}\left({}^{E}\boldsymbol{v}_{r}^{V}\right)+\begin{cases}{}^{E}\boldsymbol{\omega}_{r}^{R}\times\left({}^{E}\boldsymbol{\omega}^{R}\times\boldsymbol{r}^{VP}\right) & for \quad r=4,5,6\\ \frac{d}{dt}\left({}^{E}\boldsymbol{\omega}_{r}^{R}\right)\times\boldsymbol{r}^{VP}+{}^{E}\boldsymbol{\omega}_{r}^{R}\times\left({}^{E}\boldsymbol{\omega}^{R}\times\boldsymbol{r}^{VP}\right) & for \quad r=7,8,\ldots,12\\ 0 & otherwise\end{cases}$$

$$\frac{d}{dt}\left({}^{E}\boldsymbol{v}_{t}^{P}\right)=0$$

$$\frac{d}{dt}\left({}^{E}\boldsymbol{v}_{r}^{Q}\right)=\frac{d}{dt}\left({}^{E}\boldsymbol{v}_{r}^{P}\right)+\begin{cases}{}^{E}\boldsymbol{\omega}_{r}^{H}\times\left({}^{E}\boldsymbol{\omega}^{H}\times\boldsymbol{r}^{PQ}\right) & for \quad r=4,5,6\\ \frac{d}{dt}\left({}^{E}\boldsymbol{\omega}_{r}^{H}\right)\times\boldsymbol{r}^{PQ}+{}^{E}\boldsymbol{\omega}_{r}^{H}\times\left({}^{E}\boldsymbol{\omega}^{H}\times\boldsymbol{r}^{PQ}\right) & for \quad r=7,8,\ldots,14\\ \frac{d}{dt}\left({}^{E}\boldsymbol{\omega}_{Teet}^{H}\right)\times\boldsymbol{r}^{PQ}+{}^{E}\boldsymbol{\omega}_{Teet}^{H}\times\left({}^{E}\boldsymbol{\omega}^{H}\times\boldsymbol{r}^{PQ}\right) & for \quad r=Teet\\ 0 & otherwise\end{cases}$$

$$\frac{d}{dt}\left({}^{E}\boldsymbol{v}_{t}^{Q}\right)=0$$

$$\frac{d}{dt}\left({}^{E}\boldsymbol{v}_{r}^{C}\right)=\frac{d}{dt}\left({}^{E}\boldsymbol{v}_{r}^{Q}\right)+\begin{cases}{}^{E}\boldsymbol{\omega}_{r}^{H}\times\left({}^{E}\boldsymbol{\omega}^{H}\times\boldsymbol{r}^{QC}\right) & for\quad r=4,5,6\\ \frac{d}{dt}\left({}^{E}\boldsymbol{\omega}_{r}^{H}\right)\times\boldsymbol{r}^{QC}+{}^{E}\boldsymbol{\omega}_{r}^{H}\times\left({}^{E}\boldsymbol{\omega}^{H}\times\boldsymbol{r}^{QC}\right) & for\quad r=7,8,\ldots,14\\ \frac{d}{dt}\left({}^{E}\boldsymbol{\omega}_{Teet}^{H}\right)\times\boldsymbol{r}^{QC}+{}^{E}\boldsymbol{\omega}_{Teet}^{H}\times\left({}^{E}\boldsymbol{\omega}^{H}\times\boldsymbol{r}^{QC}\right) & for\quad r=Teet\\ 0 & otherwise\end{cases}$$

$$\frac{d}{dt}\left({}^{E}\boldsymbol{v}_{t}^{C}\right)=0$$

$$\frac{d}{dt}\left[{}^{E}\boldsymbol{v}_{r}^{S1}(r)\right]=\frac{d}{dt}\left({}^{E}\boldsymbol{v}_{r}^{Q}\right)+\begin{cases}{}^{E}\boldsymbol{\omega}_{r}^{H}\times\left[{}^{H}\boldsymbol{v}^{S1}(r)+{}^{E}\boldsymbol{\omega}^{H}\times\boldsymbol{r}^{QS1}(r)\right] & for\quad r=4,5,6\\ \frac{d}{dt}\left({}^{E}\boldsymbol{\omega}_{r}^{H}\right)\times\boldsymbol{r}^{QS1}(r)+{}^{E}\boldsymbol{\omega}_{r}^{H}\times\left[{}^{H}\boldsymbol{v}^{S1}(r)+{}^{E}\boldsymbol{\omega}^{H}\times\boldsymbol{r}^{QS1}(r)\right] & for\quad r=7,8,\ldots,14\\ -\left[S_{11}^{B1}(r)\dot{q}_{B1F1}+S_{12}^{B1}(r)\dot{q}_{B1F2}+S_{13}^{B1}(r)\dot{q}_{B1E1}\right]\boldsymbol{j}_{3}^{B1}+{}^{E}\boldsymbol{\omega}^{H}\times{}^{E}\boldsymbol{v}_{B1F1}^{S1}(r) & for\quad r=B1F1\\ -\left[S_{33}^{B1}(r)\dot{q}_{B1E1}+S_{23}^{B1}(r)\dot{q}_{B1F2}+S_{13}^{B1}(r)\dot{q}_{B1F1}\right]\boldsymbol{j}_{3}^{B1}+{}^{E}\boldsymbol{\omega}^{H}\times{}^{E}\boldsymbol{v}_{B1E1}^{S1}(r) & for\quad r=B1E1\\ -\left[S_{22}^{B1}(r)\dot{q}_{B1F2}+S_{12}^{B1}(r)\dot{q}_{B1F1}+S_{23}^{B1}(r)\dot{q}_{B1E1}\right]\boldsymbol{j}_{3}^{B1}+{}^{E}\boldsymbol{\omega}^{H}\times{}^{E}\boldsymbol{v}_{B1F2}^{S1}(r) & for\quad r=B1F2\\ \frac{d}{dt}\left({}^{E}\boldsymbol{\omega}_{Teet}^{H}\right)\times\boldsymbol{r}^{QS1}(r)+{}^{E}\boldsymbol{\omega}_{Teet}^{H}\times\left[{}^{H}\boldsymbol{v}^{S1}(r)+{}^{E}\boldsymbol{\omega}^{H}\times\boldsymbol{r}^{QS1}(r)\right] & for\quad r=Teet\\ 0 & otherwise\end{cases}$$

$$\frac{d}{dt}\left[{}^{E}\boldsymbol{v}_{t}^{S1}(r)\right]=0$$

The equations for $\frac{d}{dt}\left[{}^{E}\boldsymbol{v}_{r}^{S2}(r)\right]$ and $\frac{d}{dt}\left[{}^{E}\boldsymbol{v}_{t}^{S2}(r)\right]$ are similar.

$$\frac{d}{dt}\left({}^E\boldsymbol{v}_r^W\right)=\frac{d}{dt}\left({}^E\boldsymbol{v}_r^O\right)+\begin{cases}{}^E\boldsymbol{\omega}_r^N\times\left({}^E\boldsymbol{\omega}^N\times\boldsymbol{r}^{OW}\right) & for\quad r=4,5,6\\ \frac{d}{dt}\left({}^E\boldsymbol{\omega}_r^N\right)\times\boldsymbol{r}^{OW}+{}^E\boldsymbol{\omega}_r^N\times\left({}^E\boldsymbol{\omega}^N\times\boldsymbol{r}^{OW}\right) & for\quad r=7,8,\ldots,11\\ 0 & otherwise\end{cases}$$

$$\frac{d}{dt}\left({}^E\boldsymbol{v}_t^W\right)=0$$

$$\frac{d}{dt}\left({}^E\boldsymbol{v}_r^I\right)=\frac{d}{dt}\left({}^E\boldsymbol{v}_r^W\right)+\begin{cases}{}^E\boldsymbol{\omega}_r^A\times\left({}^E\boldsymbol{\omega}^A\times\boldsymbol{r}^{WI}\right) & for\quad r=4,5,6\\ \frac{d}{dt}\left({}^E\boldsymbol{\omega}_r^A\right)\times\boldsymbol{r}^{WI}+{}^E\boldsymbol{\omega}_r^A\times\left({}^E\boldsymbol{\omega}^A\times\boldsymbol{r}^{WI}\right) & for\quad r=7,8,\ldots,11\\ \frac{d}{dt}\left({}^E\boldsymbol{\omega}_{TFrl}^A\right)\times\boldsymbol{r}^{WI}+{}^E\boldsymbol{\omega}_{TFrl}^A\times\left({}^E\boldsymbol{\omega}^A\times\boldsymbol{r}^{WI}\right) & for\quad r=TFrl\\ 0 & otherwise\end{cases}$$

$$\frac{d}{dt}\left({}^E\boldsymbol{v}_t^I\right)=0$$

$$\frac{d}{dt}\left({}^E\boldsymbol{v}_r^J\right)=\frac{d}{dt}\left({}^E\boldsymbol{v}_r^W\right)+\begin{cases}{}^E\boldsymbol{\omega}_r^A\times\left({}^E\boldsymbol{\omega}^A\times\boldsymbol{r}^{WJ}\right) & for\quad r=4,5,6\\ \frac{d}{dt}\left({}^E\boldsymbol{\omega}_r^A\right)\times\boldsymbol{r}^{WJ}+{}^E\boldsymbol{\omega}_r^A\times\left({}^E\boldsymbol{\omega}^A\times\boldsymbol{r}^{WJ}\right) & for\quad r=7,8,\ldots,11\\ \frac{d}{dt}\left({}^E\boldsymbol{\omega}_{TFrl}^A\right)\times\boldsymbol{r}^{WJ}+{}^E\boldsymbol{\omega}_{TFrl}^A\times\left({}^E\boldsymbol{\omega}^A\times\boldsymbol{r}^{WJ}\right) & for\quad r=TFrl\\ 0 & otherwise\end{cases}$$

$$\frac{d}{dt}\left({}^E\boldsymbol{v}_t^J\right)=0$$

$$\frac{d}{dt}\left({}^E\boldsymbol{v}_r^K\right)=\frac{d}{dt}\left({}^E\boldsymbol{v}_r^W\right)+\begin{cases}{}^E\boldsymbol{\omega}_r^A\times\left({}^E\boldsymbol{\omega}^A\times\boldsymbol{r}^{WK}\right) & for \quad r=4,5,6\\ \frac{d}{dt}\left({}^E\boldsymbol{\omega}_r^A\right)\times\boldsymbol{r}^{WK}+{}^E\boldsymbol{\omega}_r^A\times\left({}^E\boldsymbol{\omega}^A\times\boldsymbol{r}^{WK}\right) & for \quad r=7,8,\ldots,11\\ \frac{d}{dt}\left({}^E\boldsymbol{\omega}_{TFrl}^A\right)\times\boldsymbol{r}^{WK}+{}^E\boldsymbol{\omega}_{TFrl}^A\times\left({}^E\boldsymbol{\omega}^A\times\boldsymbol{r}^{WK}\right) & for \quad r=TFrl\\ 0 & otherwise\end{cases}$$

$$\frac{d}{dt}\left({}^E\boldsymbol{v}_t^K\right)=0$$