2

The following are derivations of the output motions available in FAST for a 2-bladed turbine configuration. The motions for a 3-bladed turbine are ¹ very similar. Note that some of the motions are given multiple names in order to support variation among the user's preferences.

Blade 1 Tip Motions: $OoPDefl \ l = TipDxc \ l = \left\lceil r^{QS1} \left(BldFlexL \right) - TipRad \ j_3^{B1} \right\rceil \cdot i_1^{B1}$ Blade 1 OoP tip deflection (relative to rotor) (directed along the xc1-axis), (m) $IPDefl = TipDyc l = \left[r^{QSI} (BldFlexL) - TipRadj_{3}^{BI} \right] \cdot i_{2}^{BI}$ Blade 1 IP tip deflection (relative to rotor) (directed along the yc1-axis), (m) $TipDxbl = \left[r^{QSl} \left(BldFlexL \right) - TipRadj_{3}^{Bl} \right] \cdot j_{1}^{Bl}$ Blade 1 flapwise tip deflection (relative to rotor) (directed along the xb1-axis), (m) $TipDybl = \left\lceil r^{QSl} \left(BldFlexL \right) - TipRadj_{3}^{Bl} \right\rceil \cdot j_{2}^{Bl}$ Blade 1 edgewise tip deflection (relative to rotor) (directed along the yb1-axis), (m) $TipDzcl = TipDzbl = \left[r^{QSl} \left(BldFlexL \right) - TipRadj_{3}^{Bl} \right] \cdot i_{3}^{Bl} = \left[r^{QSl} \left(BldFlexL \right) - TipRadj_{3}^{Bl} \right] \cdot j_{3}^{Bl}$ Blade 1 axial tip deflection (relative to rotor) (directed along the zc1-/zb1-axis), (m) $TipALxbl = {}^{E}a^{Sl} (BldFlexL) \cdot n_{l}^{Bl} (BldFlexL)$ Blade 1 flapwise tip acceleration (absolute) (directed along the xb1-axis), (m/sec²) $TipALybl = {}^{E}a^{Sl} (BldFlexL) \cdot n_{2}^{Bl} (BldFlexL)$ Blade 1 edgewise tip acceleration (absolute) (directed along the vb1-axis), (m/sec²) $TipALzbl = {}^{E}a^{Sl} (BldFlexL) \cdot n_{3}^{Bl} (BldFlexL)$ Blade 1 axial tip acceleration (absolute) (directed along the zc1-/zb1-axis), (m/sec²) $RollDefl \, I = TipRDxb \, I = \left(\frac{180}{\pi}\right)^{H} \boldsymbol{\theta}^{MI} \left(BldFlexL\right) \cdot \boldsymbol{j}_{1}^{BI}$ Blade 1 roll tip deflection (relative to the undeflected position), (about the xb1-axis), (deg) $PtchDefl \, I = TipRDyb \, I = \left(\frac{180}{\pi}\right)^{H} \boldsymbol{\theta}^{MI} \left(BldFlexL\right) \cdot \boldsymbol{j}_{2}^{BI}$ Blade 1 pitch tip deflection (relative to the undeflected position), (about the yb1-axis), (deg) where: ${}^{H}\theta^{MI}(BldFlexL) = {}^{E}\omega^{MI}_{BIFI}(BldFlexL)q_{BIFI} + {}^{E}\omega^{MI}_{BIEI}(BldFlexL)q_{BIEI} + {}^{E}\omega^{MI}_{BIF2}(BldFlexL)q_{BIF2}$ *TipClrnc1* = *TwrClrnc1* = *Tip2Twr1* = Δ $\sqrt{\left[r^{osi}\left(BldFlexL\right)\cdot d_{1}\right]^{2} + \left[r^{osi}\left(BldFlexL\right)\cdot d_{2}\right]^{2} + \left[r^{osi}\left(BldFlexL\right)\cdot d_{3}\right]^{2}} \quad for \ r^{osi}\left(BldFlexL\right)\cdot d_{2} > 0$ Blade 1 tip-to-tower clearance, (m) $\sqrt{\left[r^{os1}\left(BldFlexL\right)\cdot d_{1}\right]^{2}+\left[r^{os1}\left(BldFlexL\right)\cdot d_{3}\right]^{2}}$ otherwise where: $\mathbf{r}^{OSI}(BldFlexL) = \mathbf{r}^{OV} + \mathbf{r}^{VP} + \mathbf{r}^{PQ} + \mathbf{r}^{QSI}(BldFlexL)$ 5

Blade 1 Local Span Motions:

$SpniALxbl = {}^{E} a^{Sl} (R^{Spani}) \cdot n_{I}^{Bl} (R^{Spani})$	1
$1,2,,5), (m/sec^2)$	
$SpniALybl = {}^{E} a^{Sl} (R^{Spani}) \cdot n_{2}^{Bl} (R^{Spani})$	
$1,2,,5), (m/sec^2)$	
$SpniALzbl = {}^{E} a^{SI} (R^{Spani}) \cdot n_{3}^{BI} (R^{Spani})$	
$1,2,,5), (m/sec^2)$	

Blade 1 local flapwise acceleration (absolute) of span station i (directed along the *local* xb1-axis) (i = 2

Blade 1 local edgewise acceleration (absolute) of span station i (directed along the *local* yb1-axis) (i =

Blade 1 axial acceleration (absolute) of span station *i* (directed along the zc1-/zb1-/local zb1-axis) (*i* = 3

<u>Blade 2 Tip Motions:</u> The output motions of blade 2 are similar to those of blade 1.

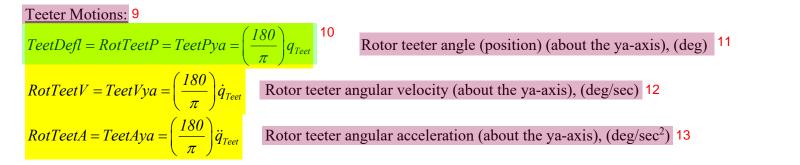
Blade Pitch Motions:

$$\begin{bmatrix} BldPitch1 = PtchPMzc1 = PtchPMzb1 = \left(\frac{180}{\pi}\right)BlPitch(1) \\ /minus zb1-axis), (deg) \\ BldPitch2 = PtchPMzc2 = PtchPMzb2 = \left(\frac{180}{\pi}\right)BlPitch(2) \\ /minus zb2 axis), (deg) \\ \end{bmatrix}$$

Blade 1 pitch angle (position) (positive towards feather / about the minus zc1-

Blade 2 pitch angle (position) (positive towards feather / about the minus zc2-8

/minus zb2-axis), (deg)



1

Shaft Motions:

Azimuth = LSSTipP = LSSTipPxa = LSSTipPxs = MOD
$$\left[\left(\frac{180}{\pi} \right) (q_{lr/r} + q_{Getz}) + AzimB1Up + 90,360 \right]$$
Rotor azimuth angle (position) (about the xa-/xs-axis), (deg)RotSpeed = LSSTipV = LSSTipVxa = LSSTipVxa = $\left(\frac{60}{2\pi} \right) (\dot{q}_{0r/r} + \dot{q}_{Getz})$ Rotor azimuth speed / angular velocity (about the xa-/xs-axis), (rpm)RotAccel = LSSTipA = LSSTipAxa = LSSTipAxa = $\left(\frac{180}{\pi} \right) (\ddot{q}_{Dr/r} + \ddot{q}_{Getz})$ Rotor azimuth angular acceleration (about the xa-/xs-axis), (deg/sec²)LSSGagP = LSSGagPxa = LSSGagPxs = MOD $\left[\left(\frac{180}{\pi} \right) q_{Getz} + AzimB1Up + 90,360 \right]$ Low-speed shaft strain gage azimuth angle (position) (onthe gearbox side of the low-speed shaft) (about the xa-/xs-axis), (deg)Low-speed shaft strain gage angular velocity (on the gearbox side of the low-speed shaft)(about the xa-/xs-axis), (rpm)LSSGagA = LSSGagAxa = LSSGagAxs = $\left(\frac{180}{2\pi} \right) \dot{q}_{Getz}$ Low-speed shaft strain gage angular velocity (on the gearbox side of the low-speed shaft)(about the xa-/xs-axis), (rpm)LSSGagA = LSSGagAxa = $\left(\frac{180}{2\pi} \right) \ddot{q}_{Getz}$ Low-speed shaft strain gage angular acceleration (on the gearbox side of the low-speed shaft)(about the xa-/xs-axis), (rpm)LSSGagA = LSSGagAxa = $\left(\frac{180}{2\pi} \right) \ddot{q}_{Getz}$ Low-speed shaft strain gage angular acceleration (on the gearbox side of the low-speed shaft)(about the xa-/xs-axis), (deg/sec²)High-speed shaft speed / angular velocity, (rpm)HSShft $= \left(\frac{60}{2\pi} \right) GBRatio \cdot \dot{q}_{Getz}$ High-speed shaft angular acceleration, (deg/sec²)Tip Sped ratio, (-)Tip speed ratio, (-)

Nacelle IMU Motions: $NcIMUTVxs = {}^{E}v^{IMU} \cdot c_{I}$ $NcIMUTVys = -{}^{E}v^{IMU} \cdot c_{3}$ $NcIMUTVzs = {}^{E}v^{IMU} \cdot c$ $NcIMUTAxs = {}^{E}a^{IMU} \cdot c_{I}$ $NcIMUTAys = -{}^{E}a^{IMU} \cdot c_{3}$ $NcIMUTAzs = {}^{E}a^{IMU} \cdot c_{2}$ $NcIMURVxs = \left(\frac{180}{\pi}\right)^{E} \boldsymbol{\omega}^{R} \cdot \boldsymbol{c}_{I}$ $NcIMURVys = -\left(\frac{180}{\pi}\right)^{E} \boldsymbol{\omega}^{R} \cdot \boldsymbol{c}_{3}$ $NcIMURVzs = \left(\frac{180}{\pi}\right)^{E} \boldsymbol{\omega}^{R} \cdot \boldsymbol{c}_{2}$ $NcIMURAxs = \left(\frac{180}{\pi}\right)^{E} \boldsymbol{\alpha}^{R} \cdot \boldsymbol{c}_{I}$ $NcIMURAys = -\left(\frac{180}{\pi}\right)^{E} \alpha^{R} \cdot c_{3}$ $NcIMURAzs = \left(\frac{180}{2}\right)^{E} \boldsymbol{\alpha}^{R} \cdot \boldsymbol{c}_{2}$

Rotor-Furl Motions: 2

$$RotFurl = RotFurlP = \left(\frac{180}{\pi}\right)$$
$$RotFurlV = \left(\frac{180}{\pi}\right)\dot{q}_{RFrl}$$
$$RotFurlA = \left(\frac{180}{\pi}\right)\ddot{q}_{RFrl}$$

Nacelle IMU translational velocity (directed along the xs-axis), (m/sec) 4 Nacelle IMU translational velocity (directed along the ys-axis), (m/sec) Nacelle IMU translational velocity (directed along the zs-axis), (m/sec) Nacelle IMU translational acceleration (directed along the xs-axis), (m/sec²) Nacelle IMU translational acceleration (directed along the ys-axis), (m/sec²) Nacelle IMU translational acceleration (directed along the zs-axis), (m/sec²) Nacelle IMU angular (rotational) velocity (about the xs-axis), (deg/sec) 5 Nacelle IMU angular (rotational) velocity (about the ys-axis), (deg/sec) 6 Nacelle IMU angular (rotational) velocity (about the zs-axis), (deg/sec) Nacelle IMU angular (rotational) acceleration (about the xs-axis), (deg/sec²) Nacelle IMU angular (rotational) acceleration (about the ys-axis), (deg/sec²) Nacelle IMU angular (rotational) acceleration (about the zs-axis), (deg/sec²)

Rotor-furl angle (position) (about the rotor-furl axis), (deg) 7

Rotor-furl angular velocity (about the rotor-furl axis), (deg/sec) 8 Rotor-furl angular acceleration (about the rotor-furl axis), (deg/sec²) 9

Yaw Motions:

 $Nac Yaw = Nac YawP = YawPzn = YawPzp = \left(\frac{180}{\pi}\right)q_{Yaw}$ $Nac YawV = YawVzn = YawVzp = \left(\frac{180}{\pi}\right)\dot{q}_{Yaw}$ $Nac YawA = YawAzn = YawAzp = \left(\frac{180}{\pi}\right)\ddot{q}_{Yaw}$ Nac YawErr = Hor WndDir - Nac Yaw - YawBrRDzt - Ptfm Yaw

Tower-Top Motions: 3

 $YawBrTDxp = \left[r^{zo} - (TowerHt + Ptfm \, Re \, f) a_2 \right] \cdot b_1^{5}$ (directed along the xp-axis), (m) $YawBrTDyp = -\left[\mathbf{r}^{\mathbf{z}o} - (TowerHt + Ptfm \, Re \, f) \mathbf{a}_2 \right] \cdot \mathbf{b}_3$ (directed along the yp-axis), (m) $YawBrTDzp = \left[r^{zo} - (TowerHt + Ptfm \, Re \, f) a_2 \right] \cdot b_2$ (directed along the zp-axis), (m) $TTDspFA = YawBrTDxt = \left[\mathbf{r}^{\mathbf{z}\mathbf{o}} - (TowerHt + Ptfm \, Re \, f) \mathbf{a}_2 \right] \cdot \mathbf{a}_1$ position) (directed along the xt-axis), (m) $TTDspSS = YawBrTDyt = -\left[r^{zo} - (TowerHt + Ptfm Re f)a_2\right] \cdot a_3$ undeflected position) (directed along the yt-axis), (m) $TTDspAx = YawBrTDzt = \left[r^{zo} - (TowerHt + Ptfm \, Re \, f) a_2 \right] \cdot a_2$ position) (directed along the zt-axis), (m) $YawBrTAxp = {^{E}a^{O}} \cdot b_{I} \quad 9$ $YawBrTAvp = -{}^{E}a^{O} \cdot b_{A}$ $YawBrTAzp = {}^{E}a^{O} \cdot b_{2}$ $TTDspRoll = YawBrRDxt = \left(\frac{180}{2}\right)^{X} \theta^{B} \cdot a_{I}$

Nacelle yaw angle (position) (about the zn-/zp-axis), (deg) Nacelle yaw angular velocity (about the zn-/zp-axis), (deg/sec) Nacelle yaw angular acceleration (about the zn-/zp-axis), (deg/sec²) Nacelle yaw error (about the zt-axis), (deg) 4

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2

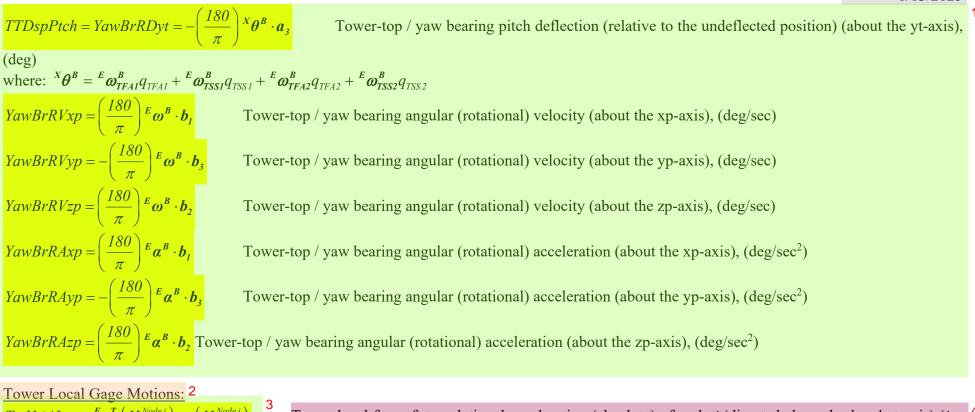
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Tower-top / yaw bearing translational deflection (relative to undeflected position)(m) $erHt + Ptfm Re f)a_2] \cdot b_3$ (m) $Ht + Ptfm Re f)a_2] \cdot b_2$ Tower-top / yaw bearing translational deflection (relative to undeflected position)(m) $Ht + Ptfm Re f)a_2] \cdot b_2$ Tower-top / yaw bearing translational deflection (relative to undeflected position)(m) $2^{o} - (TowerHt + Ptfm Re f)a_2] \cdot a_1$ Tower-top / yaw bearing fore-aft (translational) deflection (relative to undeflected position)(m) $2^{o} - (TowerHt + Ptfm Re f)a_2] \cdot a_1$ Tower-top / yaw bearing side-to-side (translational) deflection (relative to undeflected position)(m) $2^{o} - (TowerHt + Ptfm Re f)a_2] \cdot a_2$ Tower-top / yaw bearing side-to-side (translational) deflection (relative to undeflected position)(m) $2^{o} - (TowerHt + Ptfm Re f)a_2] \cdot a_2$ Tower-top / yaw bearing axial (translational) deflection (relative to undeflected position)(m) $2^{o} - (TowerHt + Ptfm Re f)a_2] \cdot a_2$ Tower-top / yaw bearing translational acceleration (directed along the xp-axis), (m/sec²)Tower-top / yaw bearing translational acceleration (directed along the yp-axis), (m/sec²)

Tower-top / yaw bearing translational acceleration (directed along the zp-axis), (m/sec²)

Tower-top / yaw bearing roll deflection (relative to the undeflected position) (about the xt-axis), 12

(deg) 13



 $TwHtiALxt = {}^{E} a^{T} (H^{Node i}) \cdot t_{1} (H^{Node i})$ $1,2,...,5), (m/sec^{2})$ $TwHtiALyt = -{}^{E} a^{T} (H^{Node i}) \cdot t_{3} (H^{Node i})$ $(i = 1,2,...,5), (m/sec^{2})$ $TwHtiALzt = {}^{E} a^{T} (H^{Node i}) \cdot t_{2} (H^{Node i})$ $1,2,...,5), (m/sec^{2})$

Tower local fore-aft translational acceleration (absolute) of node *i* (directed along the *local* xt-axis) (i = 4Tower local side-to-side translational acceleration (absolute) of node *i* (directed along the *local* yt-axis) 5

Tower local axial translational acceleration (absolute) of node i (directed along the *local* zt-axis) (i =

9

Tail-furl angle (position) (about the tail-furl axis), (deg)

Tail-furl angular velocity (about the tail-furl axis), (deg/sec) 6

Tail-furl angular acceleration (about the tail-furl axis), (deg/sec²) 7

Platform horizontal surge displacement (directed along the xt-axis), (m) Platform horizontal sway displacement (directed along the yt-axis), (m) Platform vertical heave displacement (directed along the zt-axis), (m) Platform horizontal surge displacement (directed along the xi-axis), (m) Platform horizontal sway displacement (directed along the yi-axis), (m) Platform vertical heave displacement (directed along the zi-axis), (m) Platform horizontal surge velocity (directed along the xt-axis), (m/sec) Platform horizontal sway velocity (directed along the yt-axis), (m/sec) Platform vertical heave velocity (directed along the zt-axis), (m/sec) Platform horizontal surge velocity (directed along the xi-axis), (m/sec) Platform horizontal sway velocity (directed along the yi-axis), (m/sec) Platform vertical heave velocity (directed along the zi-axis), (m/sec) Platform horizontal surge acceleration (directed along the xt-axis), (m/sec²) Platform horizontal sway acceleration (directed along the yt-axis), (m/sec²) Platform vertical heave acceleration (directed along the zt-axis), (m/sec²) Platform horizontal surge acceleration (directed along the xi-axis), (m/sec²) Platform horizontal sway acceleration (directed along the vi-axis), (m/sec²) Platform vertical heave acceleration (directed along the zi-axis), (m/sec²)

7

Tail-Furl Motions: 1 TailFurl = TailFurl P = $\left(\frac{180}{\pi}\right)q_T$ TailFurl V = $\left(\frac{180}{\pi}\right)\dot{q}_{TFrl}^5$ TailFurl A = $\left(\frac{180}{\pi}\right)\ddot{q}_{TFrl}^{Frl}$ 8

<u>Platform Motions:</u> <u>PtfmTDxt = $\mathbf{r}^{\mathbf{Z}} \cdot \mathbf{a}_{\mathbf{I}}$ </u>

 $PtfmTDyt = -\mathbf{r}^{\mathbf{Z}} \cdot \mathbf{a}_{3}$

 $PtfmTDzt = \mathbf{r}^{\mathbf{Z}} \cdot \mathbf{a}_{2}$

 $PtfmTVxt = {}^{E}v^{Z} \cdot a_{I}$

 $PtfmTVyt = -{}^{E}v^{Z} \cdot a_{3}$

 $PtfmTVzt = {}^{E}v^{Z} \cdot a,$

 $PtfmTAxt = {}^{E}a^{Z} \cdot a_{I}$

 $PtfmTAyt = -{}^{E}a^{Z} \cdot a_{z}$

 $PtfmTAzt = {}^{E}a^{Z} \cdot a,$

 $PtfmTVxi = \dot{q}_{s_{\alpha}}$

 $PtfmTVyi = \dot{q}_{su}$

 $PtfmTVzi = \dot{q}_{Hv}$

 $PtfmTAxi = \ddot{q}_{so}$

 $PtfmTAyi = \ddot{q}_{Sw}$

 $PtfmTAzi = \ddot{q}_{Hy}$

 $PtfmSurge = PtfmTDxi = q_{so}$

 $PtfmSway = PtfmTDyi = q_{sm}$

 $PtfmHeave = PtfmTDzi = q_{\mu}$

$$PtfmRoll = PtfmRDxi = \left(\frac{180}{\pi}\right)q,$$

$$PtfmPitch = PtfmRDyi = \left(\frac{180}{\pi}\right)q,$$

$$PtfmYaw = PtfmRDzi = \left(\frac{180}{\pi}\right)q,$$

$$PtfmRVxt = \left(\frac{180}{\pi}\right)^{E}\boldsymbol{\omega}^{X} \cdot \boldsymbol{a}_{1}$$

$$PtfmRVyt = -\left(\frac{180}{\pi}\right)^{E}\boldsymbol{\omega}^{X} \cdot \boldsymbol{a}_{2}$$

$$PtfmRVzt = \left(\frac{180}{\pi}\right)^{E}\boldsymbol{\omega}^{X} \cdot \boldsymbol{a}_{2}$$

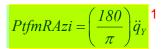
$$PtfmRVzi = \left(\frac{180}{\pi}\right)\dot{q}_{P}$$

$$PtfmRVzi = \left(\frac{180}{\pi}\right)\dot{q}_{Y}$$

$$PtfmRAxt = \left(\frac{180}{\pi}\right)^{E}\boldsymbol{\alpha}^{X} \cdot \boldsymbol{a}_{1}$$

$$PtfmRAxt = \left(\frac{180}{\pi}\right)^{E}\boldsymbol{\alpha}^{X} \cdot \boldsymbol{a}_{2}$$

Platform roll tilt displacement (about the xi-axis), (deg) Platform pitch tilt displacement (about the yi-axis), (deg) Platform yaw displacement (about the zi-axis), (deg) Platform roll tilt velocity (about the xt-axis), (deg/sec) Platform pitch tilt velocity (about the yt-axis), (deg/sec) Platform yaw velocity (about the zt-axis), (deg/sec) Platform roll tilt velocity (about the xi-axis), (deg/sec) Platform pitch tilt velocity (about the yi-axis), (deg/sec) Platform yaw velocity (about the zi-axis), (deg/sec) Platform roll tilt acceleration (about the xt-axis), (deg/sec²) Platform pitch tilt acceleration (about the yt-axis), (deg/sec²) Platform yaw acceleration (about the zt-axis), (deg/sec²) Platform roll tilt acceleration (about the xi-axis), (deg/sec²) Platform pitch tilt acceleration (about the yi-axis), (deg/sec²) Jason Jonkman FASTMotions.doc 1/13/2025



Platform yaw acceleration (about the zi-axis), (deg/sec²) 2

Tail-Furl Motions: 3		
$TFinAlpha = \left(\frac{180}{\pi}\right) TFinAOA^{4}$	Tail fin angle of attack, (deg)	
<i>TFinCLift</i> = <i>TFinCL</i>	Tail fin lift coefficient, (-)5	
<i>TFinCDrag</i> = <i>TFinCD</i>	Tail fin drag coefficient, (-)	
TFinDn Pr s = TFinQ	Tail fin dynamic pressure, (Pa)	
TFinCPFx = TFinKFx / 1,000	Tail fin tangential force, (kN)	
TFinCPFy = TFinKFy / 1,000	Tail fin normal force, (kN)	
Wind Motions: 6	\mathbf{b} beight wind value site (directed along the winewig) (\mathbf{m}/\mathbf{a})	
WindVxi = $uWind$ 7Nominal hub-height wind velocity (directed along the xi-axis), (m/s)8WindVxi = $uWind$ Cross wind hub height velocity (directed along the xi-axis) (m/s)8		
WindVyi = vWindCross-wind hub-height velocity (directed along the yi-axis), (m/s) $WindVyi = vWind$ Vertical back height relative deslated along the given in th		
	p-height wind velocity (directed along the zi-axis), (m/s)	
$TotWindV = \sqrt{WindVxi^2 + WindVyi}$	$i^{2} + WindVzi^{2}$ Total hub-height wind velocity magnitude, (m/s) ¹⁰	
$HorWindV = \sqrt{WindVxi^2 + WindVy}$	$\frac{v^2}{v^2}$ ¹¹ Horizontal hub-height wind velocity magnitude (in the xi-/yi-plane), (m/s) 12	
<i>HorWndDir</i> Horizontal hub-height wind direction (about the zi-axis), (deg) 13		
<i>VerWndDir</i> Vertical hub-height wind direction (about an axis orthogonal to the zi-axis and the horizontal wind vector), (deg)		