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There are several points on a 2-bladed turbine: Z (platform reference), Y (platform mass center), T (tower node), O (tower-top / base-plate / yaw bearing mass center), U (nacelle mass center), V (arbitrary point on rotor-furl axis), W (arbitrary point on tail-furl axis), D (center of mass of structure that furls with the rotor [not including rotor]), IMU (nacelle inertial measurement unit), P (teeter pin), SG [shaft strain gage location: i.e., a point on the shaft a distance $ShftGagL$ towards the nacelle from point P (or point Q for a 3-blader since point P does not exist)], Q (apex of coning angle), C (hub mass center), S1 (blade node for blade 1), S2 (blade node for blade 2), I (tail boom mass center), J (tail fin mass center), and K (tail fin center-of-pressure). There are also several reference frames: E (earth / inertial), X (platform / tower base), F (tower element body), B (tower-top / base plate), N (nacelle), R (structure that furls with the rotor—generator housing, etc…), L (low speed shaft on rotor end of LSS-compliance), H (hub / rotor), M1 (blade 1 element body), M2 (blade 2 element body), G (fixed in the high speed shaft / generator), and A (tail). The following are derivations of the position vectors, angular velocities, linear velocities, partial angular velocities, partial linear velocities, angular accelerations, and linear accelerations of all these points on the 2-bladed turbine (point SG's velocities and accelerations are not derived since they wont be used in the ensuing analysis). The velocities and accelerations of points on a 3-bladed turbine are very similar.

Position Vectors:

$$\boldsymbol{r}^{\boldsymbol{Z}} = q_{Sg}\boldsymbol{z_1} + q_{Hv}\boldsymbol{z_2} - q_{Sw}\boldsymbol{z_3}$$ (relative to the ground)

$$\boldsymbol{r}^{\boldsymbol{ZY}} = \left(PtfmRef - PtfmCM\right)\boldsymbol{a_2}$$

$$\begin{aligned}\boldsymbol{r}^{\boldsymbol{ZT}}(h) = {} & \left[\phi_1^{TFA}(h)q_{TFA1} + \phi_2^{TFA}(h)q_{TFA2}\right]\boldsymbol{a_1} \\ & + \left\{h + PtfmRef - TwrDraft + TwrRBHt - \frac{1}{2}\left[\begin{matrix} S_{11}^{TFA}(h)q_{TFA1}^2 + S_{22}^{TFA}(h)q_{TFA2}^2 + 2S_{12}^{TFA}(h)q_{TFA1}q_{TFA2} \\ + S_{11}^{TSS}(h)q_{TSS1}^2 + S_{22}^{TSS}(h)q_{TSS2}^2 + 2S_{12}^{TSS}(h)q_{TSS1}q_{TSS2}\end{matrix}\right]\right\}\boldsymbol{a_2} \\ & + \left[\phi_1^{TSS}(h)q_{TSS1} + \phi_2^{TSS}(h)q_{TSS2}\right]\boldsymbol{a_3}\end{aligned}$$

where,

$$S_{ij}^{TFA}(h) = \int_0^h \left[\frac{d\phi_i^{TFA}(h')}{dh'}\frac{d\phi_j^{TFA}(h')}{dh'}\right]dh' \quad (i,j = 1,2)$$ (which is symmetric)

and

$$S_{ij}^{TSS}(h) = \int_0^h \left[\frac{d\phi_i^{TSS}(h')}{dh'}\frac{d\phi_j^{TSS}(h')}{dh'}\right]dh' \quad (i,j = 1,2)$$ (which is symmetric)

Note limits on h: $0 \le h \le TowerHt + TwrDraft - TwrRBHt = TwrFlexL$

$$\boldsymbol{r}^{ZO} = \left[q_{TFA1} + q_{TFA2} \right] \boldsymbol{a}_1 + \left\{ PtfmRef + TowerHt - \frac{1}{2} \left[\begin{array}{l} S_{11}^{TFA}\left(TwrFlexL\right) q_{TFA1}^2 + S_{22}^{TFA}\left(TwrFlexL\right) q_{TFA2}^2 + 2S_{12}^{TFA}\left(TwrFlexL\right) q_{TFA1} q_{TFA2} \\ + S_{11}^{TSS}\left(TwrFlexL\right) q_{TSS1}^2 + S_{22}^{TSS}\left(TwrFlexL\right) q_{TSS2}^2 + 2S_{12}^{TSS}\left(TwrFlexL\right) q_{TSS1} q_{TSS2} \end{array} \right] \right\} \boldsymbol{a}_2 + \left[q_{TSS1} + q_{TSS2} \right] \boldsymbol{a}_3$$

$$\boldsymbol{r}^{OU} = NacCMxn\boldsymbol{d}_1 + NacCMzn\boldsymbol{d}_2 - NacCMyn\boldsymbol{d}_3$$

$$\boldsymbol{r}^{OV} = RFrlPntxn\boldsymbol{d}_1 + RFrlPntzn\boldsymbol{d}_2 - RFrlPntyn\boldsymbol{d}_3$$

$$\boldsymbol{r}^{VD} = \left(RFrlCMxn - RFrlPntxn\right)\boldsymbol{rf}_1 + \left(RFrlCMzn - RFrlPntzn\right)\boldsymbol{rf}_2 - \left(RFrlCMyn - RFrlPntyn\right)\boldsymbol{rf}_3$$

$$\boldsymbol{r}^{VIMU} = \left(NcIMUxn - RFrlPntxn\right)\boldsymbol{rf}_1 + \left(NcIMUzn - RFrlPntzn\right)\boldsymbol{rf}_2 - \left(NcIMUyn - RFrlPntyn\right)\boldsymbol{rf}_3$$

$$\boldsymbol{r}^{VP} = -RFrlPntxn\boldsymbol{rf}_1 + \left(Twr2Shft - RFrlPntzn\right)\boldsymbol{rf}_2 - \left(Yaw2Shft - RFrlPntyn\right)\boldsymbol{rf}_3 + OverHang\boldsymbol{c}_1$$

$$\boldsymbol{r}^{PSG} = ShftGagL\boldsymbol{c}_1$$

$$\boldsymbol{r}^{PQ} = -UndSling\boldsymbol{g}_1$$

$$\boldsymbol{r}^{QC} = HubCM\boldsymbol{g}_1$$

$$\boldsymbol{r}^{QS1}(r) = \left[\phi_1^{B1}(r) q_{B1F1} + \phi_2^{B1}(r) q_{B1F2} + \phi_3^{B1}(r) q_{B1E1} \right] \boldsymbol{j}_1^{B1} + \left[\psi_1^{B1}(r) q_{B1F1} + \psi_2^{B1}(r) q_{B1F2} + \psi_3^{B1}(r) q_{B1E1} \right] \boldsymbol{j}_2^{B1} + \left\{ r + HubRad - \frac{1}{2}\left[S_{11}^{B1}(r) q_{B1F1}^2 + S_{22}^{B1}(r) q_{B1F2}^2 + S_{33}^{B1}(r) q_{B1E1}^2 + 2S_{12}^{B1}(r) q_{B1F1} q_{B1F2} + 2S_{23}^{B1}(r) q_{B1F2} q_{B1E1} + 2S_{13}^{B1}(r) q_{B1F1} q_{B1E1} \right] \right\} \boldsymbol{j}_3^{B1}$$

where,

$$S_{ij}^{B1}(r) = \int_0^r \left[\frac{d\phi_i^{B1}(r')}{dr'} \frac{d\phi_j^{B1}(r')}{dr'} + \frac{d\psi_i^{B1}(r')}{dr'} \frac{d\psi_j^{B1}(r')}{dr'} \right] dr' \quad (i,j = 1,2,3)$$ (which is symmetric)

and the twisted shape functions are defined as follows:

$$\phi_1^{B1}(r) = \int_0^r \left\{ \int_0^{r'} \frac{d^2\phi_1^{B1F}(r'')}{dr''^2} cos\left[\theta_S^{B1}(r'')\right] dr'' \right\} dr',$$

$$\psi_1^{B1}(r) = -\int_0^r \left\{ \int_0^{r'} \frac{d^2\phi_1^{B1F}(r'')}{dr''^2} sin\left[\theta_S^{B1}(r'')\right] dr'' \right\} dr',$$

$$\phi_2^{B1}(r) = \int_0^r \left\{ \int_0^{r'} \frac{d^2\phi_2^{B1F}(r'')}{dr''^2} cos\left[\theta_S^{B1}(r'')\right] dr'' \right\} dr',$$

$$\psi_2^{B1}(r) = -\int_0^r \left\{ \int_0^{r'} \frac{d^2\phi_2^{B1F}(r'')}{dr''^2} sin\left[\theta_S^{B1}(r'')\right] dr'' \right\} dr',$$

$$\phi_3^{B1}(r) = \int_0^r \left\{ \int_0^{r'} \frac{d^2\phi_1^{B1E}(r'')}{dr''^2} sin\left[\theta_S^{B1}(r'')\right] dr'' \right\} dr',$$ and

$$\psi_3^{B1}(r) = \int_0^r \left\{ \int_0^{r'} \frac{d^2\phi_1^{B1E}(r'')}{dr''^2} cos\left[\theta_S^{B1}(r'')\right] dr'' \right\} dr'$$

The equation for $\boldsymbol{r}^{QS2}(r)$ is similar.

Note limit on r: $0 \le r \le TipRad - HubRad = BldFlexL$

$$\boldsymbol{r}^{OW} = TFrlPntxn\boldsymbol{d}_1 + TFrlPntzn\boldsymbol{d}_2 - TFrlPntyn\boldsymbol{d}_3$$

$$\boldsymbol{r}^{WI} = (BoomCMxn - TFrlPntxn)\boldsymbol{tf}_1 + (BoomCMzn - TFrlPntzn)\boldsymbol{tf}_2 - (BoomCMyn - TFrlPntyn)\boldsymbol{tf}_3$$

$$\boldsymbol{r}^{WJ} = (TFinCMxn - TFrlPntxn)\boldsymbol{tf}_1 + (TFinCMzn - TFrlPntzn)\boldsymbol{tf}_2 - (TFinCMyn - TFrlPntyn)\boldsymbol{tf}_3$$

$$\boldsymbol{r}^{WK} = (TFinCPxn - TFrlPntxn)\boldsymbol{tf}_1 + (TFinCPzn - TFrlPntzn)\boldsymbol{tf}_2 - (TFinCPyn - TFrlPntyn)\boldsymbol{tf}_3$$

Angular Velocities:

$${}^{E}\boldsymbol{\omega}^{X} = \dot{q}_R \boldsymbol{z_1} + \dot{q}_Y \boldsymbol{z_2} - \dot{q}_P \boldsymbol{z_3}$$

$${}^{E}\boldsymbol{\omega}^{F}(h) = {}^{E}\boldsymbol{\omega}^{X} + \left[\frac{d\phi_1^{TSS}(h)}{dh}\dot{q}_{TSS1} + \frac{d\phi_2^{TSS}(h)}{dh}\dot{q}_{TSS2}\right]\boldsymbol{a_1} - \left[\frac{d\phi_1^{TFA}(h)}{dh}\dot{q}_{TFA1} + \frac{d\phi_2^{TFA}(h)}{dh}\dot{q}_{TFA2}\right]\boldsymbol{a_3}$$

$${}^{E}\boldsymbol{\omega}^{B} = {}^{E}\boldsymbol{\omega}^{X} + \left[\left.\frac{d\phi_1^{TSS}(h)}{dh}\right|_{h=TwrFlexL}\dot{q}_{TSS1} + \left.\frac{d\phi_2^{TSS}(h)}{dh}\right|_{h=TwrFlexL}\dot{q}_{TSS2}\right]\boldsymbol{a_1} - \left[\left.\frac{d\phi_1^{TFA}(h)}{dh}\right|_{h=TwrFlexL}\dot{q}_{TFA1} + \left.\frac{d\phi_2^{TFA}(h)}{dh}\right|_{h=TwrFlexL}\dot{q}_{TFA2}\right]\boldsymbol{a_3}$$

$${}^{E}\boldsymbol{\omega}^{N} = {}^{E}\boldsymbol{\omega}^{B} + \dot{q}_{Yaw}\boldsymbol{d_2}$$

$${}^{E}\boldsymbol{\omega}^{R} = {}^{E}\boldsymbol{\omega}^{N} + \dot{q}_{RFrl}\boldsymbol{rfa}$$

where, $\boldsymbol{rfa} = cos(RFrlSkew)cos(RFrlTilt)\boldsymbol{d_1} + sin(RFrlTilt)\boldsymbol{d_2} - sin(RFrlSkew)cos(RFrlTilt)\boldsymbol{d_3}$

$${}^{E}\boldsymbol{\omega}^{L} = {}^{E}\boldsymbol{\omega}^{R} + \dot{q}_{DrTr}\boldsymbol{c_1} + \dot{q}_{GeAz}\boldsymbol{c_1}$$

$${}^{E}\boldsymbol{\omega}^{H} = {}^{E}\boldsymbol{\omega}^{L} + \dot{q}_{Teet}\boldsymbol{f_2}$$

$${}^{E}\boldsymbol{\omega}^{M1}(r) = {}^{E}\boldsymbol{\omega}^{H} - \left[\frac{d\psi_1^{B1}(r)}{dr}\dot{q}_{B1F1} + \frac{d\psi_2^{B1}(r)}{dr}\dot{q}_{B1F2} + \frac{d\psi_3^{B1}(r)}{dr}\dot{q}_{B1E1}\right]\boldsymbol{j_1^{B1}} + \left[\frac{d\phi_1^{B1}(r)}{dr}\dot{q}_{B1F1} + \frac{d\phi_2^{B1}(r)}{dr}\dot{q}_{B1F2} + \frac{d\phi_3^{B1}(r)}{dr}\dot{q}_{B1E1}\right]\boldsymbol{j_2^{B1}}$$

The equation for ${}^{E}\boldsymbol{\omega}^{M2}(r)$ is similar.

Since the generator is attached to the high speed shaft which may or may not rotate in the opposite direction of the low speed shaft and since q_{GeAz} represents the position of the low speed shaft near the entrance of the gearbox,

$${}^{E}\boldsymbol{\omega}^{G} = {}^{E}\boldsymbol{\omega}^{R} + GenDir \cdot GBRatio \cdot \dot{q}_{GeAz}\boldsymbol{c_1}$$

where,

$$GenDir = \begin{cases} -1 & for \quad GBReverse = True \\ 1 & for \quad GBReverse = False \end{cases}$$

$$ {}^{E}\boldsymbol{\omega}^{A} = {}^{E}\boldsymbol{\omega}^{N} + \dot{q}_{TFrl}\,\boldsymbol{tfa} $$

where, $\boldsymbol{tfa} = cos\left(TFrlSkew\right)cos\left(TFrlTilt\right)\boldsymbol{d_1} + sin\left(TFrlTilt\right)\boldsymbol{d_2} - sin\left(TFrlSkew\right)cos\left(TFrlTilt\right)\boldsymbol{d_3}$

Linear Velocities:

$$^{E}\boldsymbol{v}^{Z} = \dot{q}_{Sg}\boldsymbol{z_1} + \dot{q}_{Hv}\boldsymbol{z_2} - \dot{q}_{Sw}\boldsymbol{z_3}$$

$$^{E}\boldsymbol{v}^{Y} = {}^{E}\boldsymbol{v}^{Z} + {}^{E}\boldsymbol{\omega}^{X} \times \boldsymbol{r}^{ZY}$$

$$^{E}\boldsymbol{v}^{T}(h) = {}^{E}\boldsymbol{v}^{Z} + {}^{X}\boldsymbol{v}^{T}(h) + {}^{E}\boldsymbol{\omega}^{X} \times \boldsymbol{r}^{ZT}(h)$$

where,

$$\begin{aligned}
^{X}\boldsymbol{v}^{T}(h) = & \left[\phi_1^{TFA}(h)\dot{q}_{TFA1} + \phi_2^{TFA}(h)\dot{q}_{TFA2}\right]\boldsymbol{a_1} \\
& - \left[\begin{array}{l} S_{11}^{TFA}(h)q_{TFA1}\dot{q}_{TFA1} + S_{22}^{TFA}(h)q_{TFA2}\dot{q}_{TFA2} + S_{12}^{TFA}(h)\left(\dot{q}_{TFA1}q_{TFA2} + q_{TFA1}\dot{q}_{TFA2}\right) \\ + S_{11}^{TSS}(h)q_{TSS1}\dot{q}_{TSS1} + S_{22}^{TSS}(h)q_{TSS2}\dot{q}_{TSS2} + S_{12}^{TSS}(h)\left(\dot{q}_{TSS1}q_{TSS2} + q_{TSS1}\dot{q}_{TSS2}\right) \end{array}\right]\boldsymbol{a_2} \\
& + \left[\phi_1^{TSS}(h)\dot{q}_{TSS1} + \phi_2^{TSS}(h)\dot{q}_{TSS2}\right]\boldsymbol{a_3}
\end{aligned}$$

$$^{E}\boldsymbol{v}^{O} = {}^{E}\boldsymbol{v}^{Z} + {}^{X}\boldsymbol{v}^{O} + {}^{E}\boldsymbol{\omega}^{X} \times \boldsymbol{r}^{ZO}$$

where,

$$\begin{aligned}
^{X}\boldsymbol{v}^{O} = & \left[\dot{q}_{TFA1} + \dot{q}_{TFA2}\right]\boldsymbol{a_1} \\
& - \left[\begin{array}{l} S_{11}^{TFA}(TwrFlexL)q_{TFA1}\dot{q}_{TFA1} + S_{22}^{TFA}(TwrFlexL)q_{TFA2}\dot{q}_{TFA2} + S_{12}^{TFA}(TwrFlexL)\left(\dot{q}_{TFA1}q_{TFA2} + q_{TFA1}\dot{q}_{TFA2}\right) \\ + S_{11}^{TSS}(TwrFlexL)q_{TSS1}\dot{q}_{TSS1} + S_{22}^{TSS}(TwrFlexL)q_{TSS2}\dot{q}_{TSS2} + S_{12}^{TSS}(TwrFlexL)\left(\dot{q}_{TSS1}q_{TSS2} + q_{TSS1}\dot{q}_{TSS2}\right) \end{array}\right]\boldsymbol{a_2} \\
& + \left[\dot{q}_{TSS1} + \dot{q}_{TSS2}\right]\boldsymbol{a_3}
\end{aligned}$$

$$^{E}\boldsymbol{v}^{U} = {}^{E}\boldsymbol{v}^{O} + {}^{E}\boldsymbol{\omega}^{N} \times \boldsymbol{r}^{OU}$$

$$^{E}\boldsymbol{v}^{V} = {}^{E}\boldsymbol{v}^{O} + {}^{E}\boldsymbol{\omega}^{N} \times \boldsymbol{r}^{OV}$$

$$^{E}\boldsymbol{v}^{D} = {}^{E}\boldsymbol{v}^{V} + {}^{E}\boldsymbol{\omega}^{R} \times \boldsymbol{r}^{VD}$$

The equation for $^{E}\boldsymbol{v}^{IMU}$ is similar.

$$^{E}\boldsymbol{v}^{P} = {}^{E}\boldsymbol{v}^{V} + {}^{E}\boldsymbol{\omega}^{R} \times \boldsymbol{r}^{VP}$$

$$ {}^{E}\boldsymbol{v}^{Q} = {}^{E}\boldsymbol{v}^{P} + {}^{E}\boldsymbol{\omega}^{H} \times \boldsymbol{r}^{PQ} $$

$$ {}^{E}\boldsymbol{v}^{C} = {}^{E}\boldsymbol{v}^{Q} + {}^{E}\boldsymbol{\omega}^{H} \times \boldsymbol{r}^{QC} $$

$$ {}^{E}\boldsymbol{v}^{S1}(r) = {}^{E}\boldsymbol{v}^{Q} + {}^{H}\boldsymbol{v}^{S1}(r) + {}^{E}\boldsymbol{\omega}^{H} \times \boldsymbol{r}^{QS1}(r) $$

where,

$$ \begin{aligned} {}^{H}\boldsymbol{v}^{S1}(r) = {} & \left[\phi_{1}^{B1}(r)\dot{q}_{B1F1} + \phi_{2}^{B1}(r)\dot{q}_{B1F2} + \phi_{3}^{B1}(r)\dot{q}_{B1E1}\right]\boldsymbol{j}_{1}^{B1} + \left[\psi_{1}^{B1}(r)\dot{q}_{B1F1} + \psi_{2}^{B1}(r)\dot{q}_{B1F2} + \psi_{3}^{B1}(r)\dot{q}_{B1E1}\right]\boldsymbol{j}_{2}^{B1} \\ & - \left[\begin{array}{l} S_{11}^{B1}(r)q_{B1F1}\dot{q}_{B1F1} + S_{22}^{B1}(r)q_{B1F2}\dot{q}_{B1F2} + S_{33}^{B1}(r)q_{B1E1}\dot{q}_{B1E1} \\ \quad + S_{12}^{B1}(r)\left(\dot{q}_{B1F1}q_{B1F2} + q_{B1F1}\dot{q}_{B1F2}\right) + S_{23}^{B1}(r)\left(\dot{q}_{B1F2}q_{B1E1} + q_{B1F2}\dot{q}_{B1E1}\right) + S_{13}^{B1}(r)\left(\dot{q}_{B1F1}q_{B1E1} + q_{B1F1}\dot{q}_{B1E1}\right) \end{array}\right]\boldsymbol{j}_{3}^{B1} \end{aligned} $$

The equation for ${}^{E}\boldsymbol{v}^{S2}(r)$ is similar.

$$ {}^{E}\boldsymbol{v}^{W} = {}^{E}\boldsymbol{v}^{O} + {}^{E}\boldsymbol{\omega}^{N} \times \boldsymbol{r}^{OW} $$

$$ {}^{E}\boldsymbol{v}^{I} = {}^{E}\boldsymbol{v}^{W} + {}^{E}\boldsymbol{\omega}^{A} \times \boldsymbol{r}^{WI} $$

$$ {}^{E}\boldsymbol{v}^{J} = {}^{E}\boldsymbol{v}^{W} + {}^{E}\boldsymbol{\omega}^{A} \times \boldsymbol{r}^{WJ} $$

$$ {}^{E}\boldsymbol{v}^{K} = {}^{E}\boldsymbol{v}^{W} + {}^{E}\boldsymbol{\omega}^{A} \times \boldsymbol{r}^{WK} $$

Partial Angular Velocities:

Recall that: ${}^{E}\boldsymbol{\omega}^{N_i}\left(\dot{q},q,t\right)=\left(\sum_{r=1}^{22}{}^{E}\boldsymbol{\omega}_r^{N_i}\left(q,t\right)\dot{q}_r\right)+{}^{E}\boldsymbol{\omega}_t^{N_i}\left(q,t\right)$ for each rigid body N_i in the system. Note that all of the ${}^{E}\boldsymbol{\omega}_t^{N_i}$ terms are zero as will be shown.

$$
{}^{E}\boldsymbol{\omega}_r^{X}=\begin{cases}\boldsymbol{z}_1 & for \quad r=R\\ -\boldsymbol{z}_3 & for \quad r=P\\ \boldsymbol{z}_2 & for \quad r=Y\\ 0 & otherwise\end{cases}
$$

$$
{}^{E}\boldsymbol{\omega}_t^{X}=0
$$

$$
{}^{E}\boldsymbol{\omega}_r^{F}\left(h\right)={}^{E}\boldsymbol{\omega}_r^{X}+\begin{cases}-\dfrac{d\phi_1^{TFA}\left(h\right)}{dh}\boldsymbol{a}_3 & for \quad r=TFA1\\ \dfrac{d\phi_1^{TSS}\left(h\right)}{dh}\boldsymbol{a}_1 & for \quad r=TSS1\\ -\dfrac{d\phi_2^{TFA}\left(h\right)}{dh}\boldsymbol{a}_3 & for \quad r=TFA2\\ \dfrac{d\phi_2^{TSS}\left(h\right)}{dh}\boldsymbol{a}_1 & for \quad r=TSS2\\ 0 & otherwise\end{cases}
$$

$$
{}^{E}\boldsymbol{\omega}_t^{F}\left(h\right)=0
$$

$$^{E}\boldsymbol{\omega}_{r}^{B} = {}^{E}\boldsymbol{\omega}_{r}^{X} + \begin{cases} -\left.\dfrac{d\phi_{1}^{TFA}(h)}{dh}\right|_{h=TwrFlexL} \boldsymbol{a}_{3} & for \quad r = TFA1 \\ \left.\dfrac{d\phi_{1}^{TSS}(h)}{dh}\right|_{h=TwrFlexL} \boldsymbol{a}_{1} & for \quad r = TSS1 \\ -\left.\dfrac{d\phi_{2}^{TFA}(h)}{dh}\right|_{h=TwrFlexL} \boldsymbol{a}_{3} & for \quad r = TFA2 \\ \left.\dfrac{d\phi_{2}^{TSS}(h)}{dh}\right|_{h=TwrFlexL} \boldsymbol{a}_{1} & for \quad r = TSS2 \\ 0 & otherwise \end{cases}$$

$$^{E}\boldsymbol{\omega}_{t}^{B} = 0$$

$$^{E}\boldsymbol{\omega}_{r}^{N} = {}^{E}\boldsymbol{\omega}_{r}^{B} + \begin{cases} \boldsymbol{d}_{2} & for \quad r = Yaw \\ 0 & otherwise \end{cases}$$

$$^{E}\boldsymbol{\omega}_{t}^{N} = 0$$

$$^{E}\boldsymbol{\omega}_{r}^{R} = {}^{E}\boldsymbol{\omega}_{r}^{N} + \begin{cases} \boldsymbol{rfa} & for \quad r = RFrl \\ 0 & otherwise \end{cases}$$

$$^{E}\boldsymbol{\omega}_{t}^{R} = 0$$

$$^{E}\boldsymbol{\omega}_{r}^{L} = {}^{E}\boldsymbol{\omega}_{r}^{R} + \begin{cases} \boldsymbol{c}_{1} & for \quad r = GeAz \\ \boldsymbol{c}_{1} & for \quad r = DrTr \\ 0 & otherwise \end{cases}$$

$$^{E}\boldsymbol{\omega}_{t}^{L} = 0$$

$$ {}^{E}\boldsymbol{\omega}_{r}^{H} = {}^{E}\boldsymbol{\omega}_{r}^{L} + \begin{cases} \boldsymbol{f_2} & for \quad r = Teet \\ 0 & otherwise \end{cases} $$

$$ {}^{E}\boldsymbol{\omega}_{t}^{H} = 0 $$

$$ {}^{E}\boldsymbol{\omega}_{r}^{M1}(r) = {}^{E}\boldsymbol{\omega}_{r}^{H} + \begin{cases} -\frac{d\psi_{1}^{B1}(r)}{dr}\boldsymbol{j}_{1}^{B1} + \frac{d\phi_{1}^{B1}(r)}{dr}\boldsymbol{j}_{2}^{B1} & for \quad r = B1F1 \\ -\frac{d\psi_{3}^{B1}(r)}{dr}\boldsymbol{j}_{1}^{B1} + \frac{d\phi_{3}^{B1}(r)}{dr}\boldsymbol{j}_{2}^{B1} & for \quad r = B1E1 \\ -\frac{d\psi_{2}^{B1}(r)}{dr}\boldsymbol{j}_{1}^{B1} + \frac{d\phi_{2}^{B1}(r)}{dr}\boldsymbol{j}_{2}^{B1} & for \quad r = B1F2 \\ 0 & otherwise \end{cases} $$

$$ {}^{E}\boldsymbol{\omega}_{t}^{M1}(r) = 0 $$

The equations for ${}^{E}\boldsymbol{\omega}_{r}^{M2}(r)$ and ${}^{E}\boldsymbol{\omega}_{t}^{M2}(r)$ are similar.

$$ {}^{E}\boldsymbol{\omega}_{r}^{G} = {}^{E}\boldsymbol{\omega}_{r}^{R} + \begin{cases} GenDir \cdot GBRatio\boldsymbol{c_1} & for \quad r = GeAz \\ 0 & otherwise \end{cases} $$

$$ {}^{E}\boldsymbol{\omega}_{t}^{G} = 0 $$

$$ {}^{E}\boldsymbol{\omega}_{r}^{A} = {}^{E}\boldsymbol{\omega}_{r}^{N} + \begin{cases} \boldsymbol{tfa} & for \quad r = TFrl \\ 0 & otherwise \end{cases} $$

$$ {}^{E}\boldsymbol{\omega}_{t}^{A} = 0 $$

Partial Linear Velocities:

Recall that: ${}^{E}\boldsymbol{v}^{X_i}\left(\dot{q},q,t\right)=\left(\sum_{r=1}^{22}{}^{E}\boldsymbol{v}_r^{X_i}\left(q,t\right)\dot{q}_r\right)+{}^{E}\boldsymbol{v}_t^{X_i}\left(q,t\right)$ for each point X_i in the system. Note that all of the ${}^{E}\boldsymbol{v}_t^{X_i}$ terms are zero as will be shown.

$$
{}^{E}\boldsymbol{v}_r^{Z}=\begin{cases}\boldsymbol{z}_1 & for \quad r=Sg\\ -\boldsymbol{z}_3 & for \quad r=Sw\\ \boldsymbol{z}_2 & for \quad r=Hv\\ 0 & otherwise\end{cases}
$$

$$
{}^{E}\boldsymbol{v}_t^{Z}=0
$$

$$
{}^{E}\boldsymbol{v}_r^{Y}={}^{E}\boldsymbol{v}_r^{Z}+\begin{cases}{}^{E}\boldsymbol{\omega}_r^{X}\times\boldsymbol{r}^{ZY} & for \quad r=4,5,6\\ 0 & otherwise\end{cases}
$$

$$
{}^{E}\boldsymbol{v}_t^{Y}=0
$$

$$
{}^{E}\boldsymbol{v}_r^{T}\left(h\right)={}^{E}\boldsymbol{v}_r^{Z}+\begin{cases}{}^{E}\boldsymbol{\omega}_r^{X}\times\boldsymbol{r}^{ZT}\left(h\right) & for \quad r=4,5,6\\ \phi_1^{TFA}\left(h\right)\boldsymbol{a}_1-\left[S_{11}^{TFA}\left(h\right)q_{TFA1}+S_{12}^{TFA}\left(h\right)q_{TFA2}\right]\boldsymbol{a}_2 & for \quad r=TFA1\\ \phi_1^{TSS}\left(h\right)\boldsymbol{a}_3-\left[S_{11}^{TSS}\left(h\right)q_{TSS1}+S_{12}^{TSS}\left(h\right)q_{TSS2}\right]\boldsymbol{a}_2 & for \quad r=TSS1\\ \phi_2^{TFA}\left(h\right)\boldsymbol{a}_1-\left[S_{22}^{TFA}\left(h\right)q_{TFA2}+S_{12}^{TFA}\left(h\right)q_{TFA1}\right]\boldsymbol{a}_2 & for \quad r=TFA2\\ \phi_2^{TSS}\left(h\right)\boldsymbol{a}_3-\left[S_{22}^{TSS}\left(h\right)q_{TSS2}+S_{12}^{TSS}\left(h\right)q_{TSS1}\right]\boldsymbol{a}_2 & for \quad r=TSS2\\ 0 & otherwise\end{cases}
$$

$$
{}^{E}\boldsymbol{v}_t^{T}\left(h\right)=0
$$

$$
{}^{E}\boldsymbol{v}_{r}^{O} = {}^{E}\boldsymbol{v}_{r}^{Z} + \begin{cases} {}^{E}\boldsymbol{\omega}_{r}^{X} \times \boldsymbol{r}^{ZO} & for \quad r = 4,5,6 \\ \boldsymbol{a_1} - \left[S_{11}^{TFA}\left(TwrFlexL\right) q_{TFA1} + S_{12}^{TFA}\left(TwrFlexL\right) q_{TFA2} \right] \boldsymbol{a_2} & for \quad r = TFA1 \\ \boldsymbol{a_3} - \left[S_{11}^{TSS}\left(TwrFlexL\right) q_{TSS1} + S_{12}^{TSS}\left(TwrFlexL\right) q_{TSS2} \right] \boldsymbol{a_2} & for \quad r = TSS1 \\ \boldsymbol{a_1} - \left[S_{22}^{TFA}\left(TwrFlexL\right) q_{TFA2} + S_{12}^{TFA}\left(TwrFlexL\right) q_{TFA1} \right] \boldsymbol{a_2} & for \quad r = TFA2 \\ \boldsymbol{a_3} - \left[S_{22}^{TSS}\left(TwrFlexL\right) q_{TSS2} + S_{12}^{TSS}\left(TwrFlexL\right) q_{TSS1} \right] \boldsymbol{a_2} & for \quad r = TSS2 \\ 0 & otherwise \end{cases}
$$

$$
{}^{E}\boldsymbol{v}_{t}^{O} = 0
$$

$$
{}^{E}\boldsymbol{v}_{r}^{U} = {}^{E}\boldsymbol{v}_{r}^{O} + \begin{cases} {}^{E}\boldsymbol{\omega}_{r}^{N} \times \boldsymbol{r}^{OU} & for \quad r = 4,5,\ldots,11 \\ 0 & otherwise \end{cases}
$$

$$
{}^{E}\boldsymbol{v}_{t}^{U} = 0
$$

$$
{}^{E}\boldsymbol{v}_{r}^{V} = {}^{E}\boldsymbol{v}_{r}^{O} + \begin{cases} {}^{E}\boldsymbol{\omega}_{r}^{N} \times \boldsymbol{r}^{OV} & for \quad r = 4,5,\ldots,11 \\ 0 & otherwise \end{cases}
$$

$$
{}^{E}\boldsymbol{v}_{t}^{V} = 0
$$

$$
{}^{E}\boldsymbol{v}_{r}^{D} = {}^{E}\boldsymbol{v}_{r}^{V} + \begin{cases} {}^{E}\boldsymbol{\omega}_{r}^{R} \times \boldsymbol{r}^{VD} & for \quad r = 4,5,\ldots,12 \\ 0 & otherwise \end{cases}
$$

$$
{}^{E}\boldsymbol{v}_{t}^{D} = 0
$$

The equations for ${}^{E}\boldsymbol{v}_{r}^{IMU}$ and ${}^{E}\boldsymbol{v}_{t}^{IMU}$ are similar.

$$
{}^{E}\boldsymbol{v}_{r}^{P} = {}^{E}\boldsymbol{v}_{r}^{V} + \begin{cases} {}^{E}\boldsymbol{\omega}_{r}^{R} \times \boldsymbol{r}^{VP} & for \quad r = 4,5,\ldots,12 \\ 0 & otherwise \end{cases}
$$

$$
{}^{E}\boldsymbol{v}_{t}^{P} = 0
$$

$$
{}^{E}\boldsymbol{v}_{r}^{Q} = {}^{E}\boldsymbol{v}_{r}^{P} + \begin{cases} {}^{E}\boldsymbol{\omega}_{r}^{H} \times \boldsymbol{r}^{PQ} & for \quad r = 4,5,\ldots,14 \\ {}^{E}\boldsymbol{\omega}_{Teet}^{H} \times \boldsymbol{r}^{PQ} & for \quad r = Teet \\ 0 & otherwise \end{cases}
$$

$$
{}^{E}\boldsymbol{v}_{t}^{Q} = 0
$$

$$
{}^{E}\boldsymbol{v}_{r}^{C} = {}^{E}\boldsymbol{v}_{r}^{Q} + \begin{cases} {}^{E}\boldsymbol{\omega}_{r}^{H} \times \boldsymbol{r}^{QC} & for \quad r = 4,5,\ldots,14 \\ {}^{E}\boldsymbol{\omega}_{Teet}^{H} \times \boldsymbol{r}^{QC} & for \quad r = Teet \\ 0 & otherwise \end{cases}
$$

$$
{}^{E}\boldsymbol{v}_{t}^{C} = 0
$$

$$
{}^{E}\boldsymbol{v}_{r}^{S1}(r) = {}^{E}\boldsymbol{v}_{r}^{Q} + \begin{cases} {}^{E}\boldsymbol{\omega}_{r}^{H} \times \boldsymbol{r}^{QS1}(r) & for \quad r = 4,5,\ldots,14 \\ \phi_{1}^{B1}(r)\,\boldsymbol{j}_{1}^{B1} + \psi_{1}^{B1}(r)\,\boldsymbol{j}_{2}^{B1} - \left[S_{11}^{B1}(r)q_{B1F1} + S_{12}^{B1}(r)q_{B1F2} + S_{13}^{B1}(r)q_{B1E1}\right]\boldsymbol{j}_{3}^{B1} & for \quad r = B1F1 \\ \phi_{3}^{B1}(r)\,\boldsymbol{j}_{1}^{B1} + \psi_{3}^{B1}(r)\,\boldsymbol{j}_{2}^{B1} - \left[S_{33}^{B1}(r)q_{B1E1} + S_{23}^{B1}(r)q_{B1F2} + S_{13}^{B1}(r)q_{B1F1}\right]\boldsymbol{j}_{3}^{B1} & for \quad r = B1E1 \\ \phi_{2}^{B1}(r)\,\boldsymbol{j}_{1}^{B1} + \psi_{2}^{B1}(r)\,\boldsymbol{j}_{2}^{B1} - \left[S_{22}^{B1}(r)q_{B1F2} + S_{12}^{B1}(r)q_{B1F1} + S_{23}^{B1}(r)q_{B1E1}\right]\boldsymbol{j}_{3}^{B1} & for \quad r = B1F2 \\ {}^{E}\boldsymbol{\omega}_{Teet}^{H} \times \boldsymbol{r}^{QS1}(r) & for \quad r = Teet \\ 0 & otherwise \end{cases}
$$

$$
{}^{E}\boldsymbol{v}_{t}^{S1}(r) = 0
$$

The equations for ${}^{E}\boldsymbol{v}_{r}^{S2}(r)$ and ${}^{E}\boldsymbol{v}_{t}^{S2}(r)$ are similar.

$$
{}^{E}\boldsymbol{v}_{r}^{W} = {}^{E}\boldsymbol{v}_{r}^{O} + \begin{cases} {}^{E}\boldsymbol{\omega}_{r}^{N} \times \boldsymbol{r}^{OW} & for \quad r = 4,5,\ldots,11 \\ 0 & otherwise \end{cases}
$$

$$
{}^{E}\boldsymbol{v}_{t}^{W} = 0
$$

$$ {}^{E}\boldsymbol{v}_r^{I} = {}^{E}\boldsymbol{v}_r^{W} + \begin{cases} {}^{E}\boldsymbol{\omega}_r^{A} \times \boldsymbol{r}^{WI} & for \quad r = 4,5,\ldots,11 \\ {}^{E}\boldsymbol{\omega}_{TFrl}^{A} \times \boldsymbol{r}^{WI} & for \quad r = TFrl \\ 0 & otherwise \end{cases} $$

$$ {}^{E}\boldsymbol{v}_t^{I} = 0 $$

$$ {}^{E}\boldsymbol{v}_r^{J} = {}^{E}\boldsymbol{v}_r^{W} + \begin{cases} {}^{E}\boldsymbol{\omega}_r^{A} \times \boldsymbol{r}^{WJ} & for \quad r = 4,5,\ldots,11 \\ {}^{E}\boldsymbol{\omega}_{TFrl}^{A} \times \boldsymbol{r}^{WJ} & for \quad r = TFrl \\ 0 & otherwise \end{cases} $$

$$ {}^{E}\boldsymbol{v}_t^{J} = 0 $$

$$ {}^{E}\boldsymbol{v}_r^{K} = {}^{E}\boldsymbol{v}_r^{W} + \begin{cases} {}^{E}\boldsymbol{\omega}_r^{A} \times \boldsymbol{r}^{WK} & for \quad r = 4,5,\ldots,11 \\ {}^{E}\boldsymbol{\omega}_{TFrl}^{A} \times \boldsymbol{r}^{WK} & for \quad r = TFrl \\ 0 & otherwise \end{cases} $$

$$ {}^{E}\boldsymbol{v}_t^{K} = 0 $$

Angular Accelerations:

Recall that: ${}^{E}\boldsymbol{\alpha}^{N_i}\left(\ddot{q},\dot{q},q,t\right)=\left(\sum_{r=1}^{22}{}^{E}\boldsymbol{\omega}_r^{N_i}\left(q,t\right)\ddot{q}_r\right)+\left[\sum_{r=1}^{22}\frac{d}{dt}\left({}^{E}\boldsymbol{\omega}_r^{N_i}\left(q,t\right)\right)\dot{q}_r\right]+\frac{d}{dt}\left({}^{E}\boldsymbol{\omega}_t^{N_i}\left(q,t\right)\right)$ for each rigid body N_i in the system. Note that the $\frac{d}{dt}\left({}^{E}\boldsymbol{\omega}_r^{N_i}\right)$ terms are all vector functions of $\left(\dot{q},q,t\right)$ and that all of the $\frac{d}{dt}\left({}^{E}\boldsymbol{\omega}_t^{N_i}\right)$ terms are zero as will be shown.

$$\frac{d}{dt}\left({}^{E}\boldsymbol{\omega}_r^{X}\right)=0$$
$$\frac{d}{dt}\left({}^{E}\boldsymbol{\omega}_t^{X}\right)=0$$

$$\frac{d}{dt}\left[{}^{E}\boldsymbol{\omega}_r^{F}\left(h\right)\right]=\begin{cases}{}^{E}\boldsymbol{\omega}^{X}\times{}^{E}\boldsymbol{\omega}_r^{F}\left(h\right) & for\quad r=7,8,\ldots,10\\ 0 & otherwise\end{cases}$$
$$\frac{d}{dt}\left[{}^{E}\boldsymbol{\omega}_t^{F}\left(h\right)\right]=0$$

$$\frac{d}{dt}\left({}^{E}\boldsymbol{\omega}_r^{B}\right)=\begin{cases}{}^{E}\boldsymbol{\omega}^{X}\times{}^{E}\boldsymbol{\omega}_r^{B} & for\quad r=7,8,\ldots,10\\ 0 & otherwise\end{cases}$$
$$\frac{d}{dt}\left({}^{E}\boldsymbol{\omega}_t^{B}\right)=0$$

$$\frac{d}{dt}\left({}^{E}\boldsymbol{\omega}_r^{N}\right)=\frac{d}{dt}\left({}^{E}\boldsymbol{\omega}_r^{B}\right)+\begin{cases}{}^{E}\boldsymbol{\omega}^{B}\times{}^{E}\boldsymbol{\omega}_{Yaw}^{N} & for\quad r=Yaw\\ 0 & otherwise\end{cases}$$
$$\frac{d}{dt}\left({}^{E}\boldsymbol{\omega}_t^{N}\right)=0$$

$$\frac{d}{dt}\left({}^E\boldsymbol{\omega}_r^R\right)=\frac{d}{dt}\left({}^E\boldsymbol{\omega}_r^N\right)+\begin{cases}{}^E\boldsymbol{\omega}^N\times{}^E\boldsymbol{\omega}_{RFrl}^R & for \quad r=RFrl\\ 0 & otherwise\end{cases}$$

$$\frac{d}{dt}\left({}^E\boldsymbol{\omega}_t^R\right)=0$$

$$\frac{d}{dt}\left({}^E\boldsymbol{\omega}_r^L\right)=\frac{d}{dt}\left({}^E\boldsymbol{\omega}_r^R\right)+\begin{cases}{}^E\boldsymbol{\omega}^R\times{}^E\boldsymbol{\omega}_{GeAz}^L & for \quad r=GeAz\\ {}^E\boldsymbol{\omega}^R\times{}^E\boldsymbol{\omega}_{DrTr}^L & for \quad r=DrTr\\ 0 & otherwise\end{cases}$$

$$\frac{d}{dt}\left({}^E\boldsymbol{\omega}_t^L\right)=0$$

$$\frac{d}{dt}\left({}^E\boldsymbol{\omega}_r^H\right)=\frac{d}{dt}\left({}^E\boldsymbol{\omega}_r^L\right)+\begin{cases}{}^E\boldsymbol{\omega}^H\times{}^E\boldsymbol{\omega}_{Teet}^H & for \quad r=Teet\\ 0 & otherwise\end{cases}$$

$$\frac{d}{dt}\left({}^E\boldsymbol{\omega}_t^H\right)=0$$

$$\frac{d}{dt}\left[{}^E\boldsymbol{\omega}_r^{M1}(r)\right]=\frac{d}{dt}\left({}^E\boldsymbol{\omega}_r^H\right)+\begin{cases}{}^E\boldsymbol{\omega}^H\times{}^E\boldsymbol{\omega}_{B1F1}^{M1}(r) & for \quad r=B1F1\\ {}^E\boldsymbol{\omega}^H\times{}^E\boldsymbol{\omega}_{B1E1}^{M1}(r) & for \quad r=B1E1\\ {}^E\boldsymbol{\omega}^H\times{}^E\boldsymbol{\omega}_{B1F2}^{M1}(r) & for \quad r=B1F2\\ 0 & otherwise\end{cases}$$

$$\frac{d}{dt}\left[{}^E\boldsymbol{\omega}_t^{M1}(r)\right]=0$$

The equations for $\frac{d}{dt}\left[{}^E\boldsymbol{\omega}_r^{M2}(r)\right]$ and $\frac{d}{dt}\left[{}^E\boldsymbol{\omega}_t^{M2}(r)\right]$ are similar.

$$\frac{d}{dt}\left({}^{E}\boldsymbol{\omega}_{r}^{G}\right)=\frac{d}{dt}\left({}^{E}\boldsymbol{\omega}_{r}^{R}\right)+\begin{cases}{}^{E}\boldsymbol{\omega}^{R}\times{}^{E}\boldsymbol{\omega}_{GeAz}^{G} & for \quad r=GeAz\\ 0 & otherwise\end{cases}$$

$$\frac{d}{dt}\left({}^{E}\boldsymbol{\omega}_{t}^{G}\right)=0$$

$$\frac{d}{dt}\left({}^{E}\boldsymbol{\omega}_{r}^{A}\right)=\frac{d}{dt}\left({}^{E}\boldsymbol{\omega}_{r}^{N}\right)+\begin{cases}{}^{E}\boldsymbol{\omega}^{N}\times{}^{E}\boldsymbol{\omega}_{TFrl}^{A} & for \quad r=TFrl\\ 0 & otherwise\end{cases}$$

$$\frac{d}{dt}\left({}^{E}\boldsymbol{\omega}_{t}^{A}\right)=0$$

Linear Accelerations:

Recall that: ${}^{E}\boldsymbol{a}^{X_i}\left(\ddot{q},\dot{q},q,t\right)=\left(\sum_{r=1}^{22}{}^{E}\boldsymbol{v}_r^{X_i}\left(q,t\right)\ddot{q}_r\right)+\left[\sum_{r=1}^{22}\frac{d}{dt}\left({}^{E}\boldsymbol{v}_r^{X_i}\left(q,t\right)\right)\dot{q}_r\right]+\frac{d}{dt}\left({}^{E}\boldsymbol{v}_t^{X_i}\left(q,t\right)\right)$ for each point X_i in the system. Note that the $\frac{d}{dt}\left({}^{E}\boldsymbol{v}_r^{X_i}\right)$ terms are all vector functions of $\left(\dot{q},q,t\right)$ and that all of the $\frac{d}{dt}\left({}^{E}\boldsymbol{v}_t^{X_i}\right)$ terms are zero as will be shown.

$$\frac{d}{dt}\left({}^{E}\boldsymbol{v}_r^{Z}\right)=0$$

$$\frac{d}{dt}\left({}^{E}\boldsymbol{v}_t^{Z}\right)=0$$

$$\frac{d}{dt}\left({}^{E}\boldsymbol{v}_r^{Y}\right)=\begin{cases}{}^{E}\boldsymbol{\omega}_r^{X}\times\left({}^{E}\boldsymbol{\omega}^{X}\times\boldsymbol{r}^{ZY}\right) & for\quad r=4,5,6\\ 0 & otherwise\end{cases}$$

$$\frac{d}{dt}\left({}^{E}\boldsymbol{v}_t^{Y}\right)=0$$

$$\frac{d}{dt}\left[{}^{E}\boldsymbol{v}_r^{T}\left(h\right)\right]=\begin{cases}{}^{E}\boldsymbol{\omega}_r^{X}\times\left[{}^{X}\boldsymbol{v}^{T}\left(h\right)+{}^{E}\boldsymbol{\omega}^{X}\times\boldsymbol{r}^{ZT}\left(h\right)\right] & for\quad r=4,5,6\\ -\left[S_{11}^{TFA}\left(h\right)\dot{q}_{TFA1}+S_{12}^{TFA}\left(h\right)\dot{q}_{TFA1}\right]\boldsymbol{a}_2+{}^{E}\boldsymbol{\omega}^{X}\times{}^{E}\boldsymbol{v}_{TFA1}^{T}\left(h\right) & for\quad r=TFA1\\ -\left[S_{11}^{TSS}\left(h\right)\dot{q}_{TSS1}+S_{12}^{TSS}\left(h\right)\dot{q}_{TSS2}\right]\boldsymbol{a}_2+{}^{E}\boldsymbol{\omega}^{X}\times{}^{E}\boldsymbol{v}_{TSS1}^{T}\left(h\right) & for\quad r=TSS1\\ -\left[S_{22}^{TFA}\left(h\right)\dot{q}_{TFA2}+S_{12}^{TFA}\left(h\right)\dot{q}_{TFA1}\right]\boldsymbol{a}_2+{}^{E}\boldsymbol{\omega}^{X}\times{}^{E}\boldsymbol{v}_{TFA2}^{T}\left(h\right) & for\quad r=TFA2\\ -\left[S_{22}^{TSS}\left(h\right)\dot{q}_{TSS2}+S_{12}^{TSS}\left(h\right)\dot{q}_{TSS1}\right]\boldsymbol{a}_2+{}^{E}\boldsymbol{\omega}^{X}\times{}^{E}\boldsymbol{v}_{TSS2}^{T}\left(h\right) & for\quad r=TSS2\\ 0 & otherwise\end{cases}$$

$$\frac{d}{dt}\left[{}^{E}\boldsymbol{v}_t^{T}\left(h\right)\right]=0$$

$$\frac{d}{dt}\left({}^{E}\boldsymbol{v}_{r}^{O}\right)=\begin{cases}{}^{E}\boldsymbol{\omega}_{r}^{X}\times\left({}^{X}\boldsymbol{v}^{O}+{}^{E}\boldsymbol{\omega}^{X}\times\boldsymbol{r}^{ZO}\right) & for \quad r=4,5,6\\ -\left[S_{11}^{TFA}\left(TwrFlexL\right)\dot{q}_{TFA1}+S_{12}^{TFA}\left(TwrFlexL\right)\dot{q}_{TFA1}\right]\boldsymbol{a}_{2}+{}^{E}\boldsymbol{\omega}^{X}\times{}^{E}\boldsymbol{v}_{TFA1}^{O} & for \quad r=TFA1\\ -\left[S_{11}^{TSS}\left(TwrFlexL\right)\dot{q}_{TSS1}+S_{12}^{TSS}\left(TwrFlexL\right)\dot{q}_{TSS2}\right]\boldsymbol{a}_{2}+{}^{E}\boldsymbol{\omega}^{X}\times{}^{E}\boldsymbol{v}_{TSS1}^{O} & for \quad r=TSS1\\ -\left[S_{22}^{TFA}\left(TwrFlexL\right)\dot{q}_{TFA2}+S_{12}^{TFA}\left(TwrFlexL\right)\dot{q}_{TFA1}\right]\boldsymbol{a}_{2}+{}^{E}\boldsymbol{\omega}^{X}\times{}^{E}\boldsymbol{v}_{TFA2}^{O} & for \quad r=TFA2\\ -\left[S_{22}^{TSS}\left(TwrFlexL\right)\dot{q}_{TSS2}+S_{12}^{TSS}\left(TwrFlexL\right)\dot{q}_{TSS1}\right]\boldsymbol{a}_{2}+{}^{E}\boldsymbol{\omega}^{X}\times{}^{E}\boldsymbol{v}_{TSS2}^{O} & for \quad r=TSS2\\ 0 & otherwise\end{cases}$$

$$\frac{d}{dt}\left({}^{E}\boldsymbol{v}_{t}^{O}\right)=0$$

$$\frac{d}{dt}\left({}^{E}\boldsymbol{v}_{r}^{U}\right)=\frac{d}{dt}\left({}^{E}\boldsymbol{v}_{r}^{O}\right)+\begin{cases}{}^{E}\boldsymbol{\omega}_{r}^{N}\times\left({}^{E}\boldsymbol{\omega}^{N}\times\boldsymbol{r}^{OU}\right) & for \quad r=4,5,6\\ \frac{d}{dt}\left({}^{E}\boldsymbol{\omega}_{r}^{N}\right)\times\boldsymbol{r}^{OU}+{}^{E}\boldsymbol{\omega}_{r}^{N}\times\left({}^{E}\boldsymbol{\omega}^{N}\times\boldsymbol{r}^{OU}\right) & for \quad r=7,8,\ldots,11\\ 0 & otherwise\end{cases}$$

$$\frac{d}{dt}\left({}^{E}\boldsymbol{v}_{t}^{U}\right)=0$$

$$\frac{d}{dt}\left({}^{E}\boldsymbol{v}_{r}^{V}\right)=\frac{d}{dt}\left({}^{E}\boldsymbol{v}_{r}^{O}\right)+\begin{cases}{}^{E}\boldsymbol{\omega}_{r}^{N}\times\left({}^{E}\boldsymbol{\omega}^{N}\times\boldsymbol{r}^{OV}\right) & for \quad r=4,5,6\\ \frac{d}{dt}\left({}^{E}\boldsymbol{\omega}_{r}^{N}\right)\times\boldsymbol{r}^{OV}+{}^{E}\boldsymbol{\omega}_{r}^{N}\times\left({}^{E}\boldsymbol{\omega}^{N}\times\boldsymbol{r}^{OV}\right) & for \quad r=7,8,\ldots,11\\ 0 & otherwise\end{cases}$$

$$\frac{d}{dt}\left({}^{E}\boldsymbol{v}_{t}^{V}\right)=0$$

$$\frac{d}{dt}\left({}^E\boldsymbol{v}_r^D\right)=\frac{d}{dt}\left({}^E\boldsymbol{v}_r^V\right)+\begin{cases}{}^E\boldsymbol{\omega}_r^R\times\left({}^E\boldsymbol{\omega}^R\times\boldsymbol{r}^{VD}\right) & for \quad r=4,5,6\\ \frac{d}{dt}\left({}^E\boldsymbol{\omega}_r^R\right)\times\boldsymbol{r}^{VD}+{}^E\boldsymbol{\omega}_r^R\times\left({}^E\boldsymbol{\omega}^R\times\boldsymbol{r}^{VD}\right) & for \quad r=7,8,\ldots,12\\ 0 & otherwise\end{cases}$$

$$\frac{d}{dt}\left({}^E\boldsymbol{v}_t^D\right)=0$$

The equations for $\frac{d}{dt}\left({}^E\boldsymbol{v}_r^{IMU}\right)$ and $\frac{d}{dt}\left({}^E\boldsymbol{v}_t^{IMU}\right)$ are similar.

$$\frac{d}{dt}\left({}^E\boldsymbol{v}_r^P\right)=\frac{d}{dt}\left({}^E\boldsymbol{v}_r^V\right)+\begin{cases}{}^E\boldsymbol{\omega}_r^R\times\left({}^E\boldsymbol{\omega}^R\times\boldsymbol{r}^{VP}\right) & for \quad r=4,5,6\\ \frac{d}{dt}\left({}^E\boldsymbol{\omega}_r^R\right)\times\boldsymbol{r}^{VP}+{}^E\boldsymbol{\omega}_r^R\times\left({}^E\boldsymbol{\omega}^R\times\boldsymbol{r}^{VP}\right) & for \quad r=7,8,\ldots,12\\ 0 & otherwise\end{cases}$$

$$\frac{d}{dt}\left({}^E\boldsymbol{v}_t^P\right)=0$$

$$\frac{d}{dt}\left({}^E\boldsymbol{v}_r^Q\right)=\frac{d}{dt}\left({}^E\boldsymbol{v}_r^P\right)+\begin{cases}{}^E\boldsymbol{\omega}_r^H\times\left({}^E\boldsymbol{\omega}^H\times\boldsymbol{r}^{PQ}\right) & for \quad r=4,5,6\\ \frac{d}{dt}\left({}^E\boldsymbol{\omega}_r^H\right)\times\boldsymbol{r}^{PQ}+{}^E\boldsymbol{\omega}_r^H\times\left({}^E\boldsymbol{\omega}^H\times\boldsymbol{r}^{PQ}\right) & for \quad r=7,8,\ldots,14\\ \frac{d}{dt}\left({}^E\boldsymbol{\omega}_{Teet}^H\right)\times\boldsymbol{r}^{PQ}+{}^E\boldsymbol{\omega}_{Teet}^H\times\left({}^E\boldsymbol{\omega}^H\times\boldsymbol{r}^{PQ}\right) & for \quad r=Teet\\ 0 & otherwise\end{cases}$$

$$\frac{d}{dt}\left({}^E\boldsymbol{v}_t^Q\right)=0$$

$$\frac{d}{dt}\left({}^{E}\boldsymbol{v}_{r}^{C}\right)=\frac{d}{dt}\left({}^{E}\boldsymbol{v}_{r}^{Q}\right)+\begin{cases}{}^{E}\boldsymbol{\omega}_{r}^{H}\times\left({}^{E}\boldsymbol{\omega}^{H}\times\boldsymbol{r}^{QC}\right) & for\quad r=4,5,6\\ \frac{d}{dt}\left({}^{E}\boldsymbol{\omega}_{r}^{H}\right)\times\boldsymbol{r}^{QC}+{}^{E}\boldsymbol{\omega}_{r}^{H}\times\left({}^{E}\boldsymbol{\omega}^{H}\times\boldsymbol{r}^{QC}\right) & for\quad r=7,8,\ldots,14\\ \frac{d}{dt}\left({}^{E}\boldsymbol{\omega}_{Teet}^{H}\right)\times\boldsymbol{r}^{QC}+{}^{E}\boldsymbol{\omega}_{Teet}^{H}\times\left({}^{E}\boldsymbol{\omega}^{H}\times\boldsymbol{r}^{QC}\right) & for\quad r=Teet\\ 0 & otherwise\end{cases}$$

$$\frac{d}{dt}\left({}^{E}\boldsymbol{v}_{t}^{C}\right)=0$$

$$\frac{d}{dt}\left[{}^{E}\boldsymbol{v}_{r}^{S1}(r)\right]=\frac{d}{dt}\left({}^{E}\boldsymbol{v}_{r}^{Q}\right)+\begin{cases}{}^{E}\boldsymbol{\omega}_{r}^{H}\times\left[{}^{H}\boldsymbol{v}^{S1}(r)+{}^{E}\boldsymbol{\omega}^{H}\times\boldsymbol{r}^{QS1}(r)\right] & for\quad r=4,5,6\\ \frac{d}{dt}\left({}^{E}\boldsymbol{\omega}_{r}^{H}\right)\times\boldsymbol{r}^{QS1}(r)+{}^{E}\boldsymbol{\omega}_{r}^{H}\times\left[{}^{H}\boldsymbol{v}^{S1}(r)+{}^{E}\boldsymbol{\omega}^{H}\times\boldsymbol{r}^{QS1}(r)\right] & for\quad r=7,8,\ldots,14\\ -\left[S_{11}^{B1}(r)\dot{q}_{B1F1}+S_{12}^{B1}(r)\dot{q}_{B1F2}+S_{13}^{B1}(r)\dot{q}_{B1E1}\right]\boldsymbol{j}_{3}^{B1}+{}^{E}\boldsymbol{\omega}^{H}\times{}^{E}\boldsymbol{v}_{B1F1}^{S1}(r) & for\quad r=B1F1\\ -\left[S_{33}^{B1}(r)\dot{q}_{B1E1}+S_{23}^{B1}(r)\dot{q}_{B1F2}+S_{13}^{B1}(r)\dot{q}_{B1F1}\right]\boldsymbol{j}_{3}^{B1}+{}^{E}\boldsymbol{\omega}^{H}\times{}^{E}\boldsymbol{v}_{B1E1}^{S1}(r) & for\quad r=B1E1\\ -\left[S_{22}^{B1}(r)\dot{q}_{B1F2}+S_{12}^{B1}(r)\dot{q}_{B1F1}+S_{23}^{B1}(r)\dot{q}_{B1E1}\right]\boldsymbol{j}_{3}^{B1}+{}^{E}\boldsymbol{\omega}^{H}\times{}^{E}\boldsymbol{v}_{B1F2}^{S1}(r) & for\quad r=B1F2\\ \frac{d}{dt}\left({}^{E}\boldsymbol{\omega}_{Teet}^{H}\right)\times\boldsymbol{r}^{QS1}(r)+{}^{E}\boldsymbol{\omega}_{Teet}^{H}\times\left[{}^{H}\boldsymbol{v}^{S1}(r)+{}^{E}\boldsymbol{\omega}^{H}\times\boldsymbol{r}^{QS1}(r)\right] & for\quad r=Teet\\ 0 & otherwise\end{cases}$$

$$\frac{d}{dt}\left[{}^{E}\boldsymbol{v}_{t}^{S1}(r)\right]=0$$

The equations for $\frac{d}{dt}\left[{}^{E}\boldsymbol{v}_{r}^{S2}(r)\right]$ and $\frac{d}{dt}\left[{}^{E}\boldsymbol{v}_{t}^{S2}(r)\right]$ are similar.

$$\frac{d}{dt}\left({}^{E}\boldsymbol{v}_r^{W}\right)=\frac{d}{dt}\left({}^{E}\boldsymbol{v}_r^{O}\right)+\begin{cases}{}^{E}\boldsymbol{\omega}_r^{N}\times\left({}^{E}\boldsymbol{\omega}^{N}\times\boldsymbol{r}^{OW}\right) & for\quad r=4,5,6\\ \frac{d}{dt}\left({}^{E}\boldsymbol{\omega}_r^{N}\right)\times\boldsymbol{r}^{OW}+{}^{E}\boldsymbol{\omega}_r^{N}\times\left({}^{E}\boldsymbol{\omega}^{N}\times\boldsymbol{r}^{OW}\right) & for\quad r=7,8,\ldots,11\\ 0 & otherwise\end{cases}$$

$$\frac{d}{dt}\left({}^{E}\boldsymbol{v}_t^{W}\right)=0$$

$$\frac{d}{dt}\left({}^{E}\boldsymbol{v}_r^{I}\right)=\frac{d}{dt}\left({}^{E}\boldsymbol{v}_r^{W}\right)+\begin{cases}{}^{E}\boldsymbol{\omega}_r^{A}\times\left({}^{E}\boldsymbol{\omega}^{A}\times\boldsymbol{r}^{WI}\right) & for\quad r=4,5,6\\ \frac{d}{dt}\left({}^{E}\boldsymbol{\omega}_r^{A}\right)\times\boldsymbol{r}^{WI}+{}^{E}\boldsymbol{\omega}_r^{A}\times\left({}^{E}\boldsymbol{\omega}^{A}\times\boldsymbol{r}^{WI}\right) & for\quad r=7,8,\ldots,11\\ \frac{d}{dt}\left({}^{E}\boldsymbol{\omega}_{TFrl}^{A}\right)\times\boldsymbol{r}^{WI}+{}^{E}\boldsymbol{\omega}_{TFrl}^{A}\times\left({}^{E}\boldsymbol{\omega}^{A}\times\boldsymbol{r}^{WI}\right) & for\quad r=TFrl\\ 0 & otherwise\end{cases}$$

$$\frac{d}{dt}\left({}^{E}\boldsymbol{v}_t^{I}\right)=0$$

$$\frac{d}{dt}\left({}^{E}\boldsymbol{v}_r^{J}\right)=\frac{d}{dt}\left({}^{E}\boldsymbol{v}_r^{W}\right)+\begin{cases}{}^{E}\boldsymbol{\omega}_r^{A}\times\left({}^{E}\boldsymbol{\omega}^{A}\times\boldsymbol{r}^{WJ}\right) & for\quad r=4,5,6\\ \frac{d}{dt}\left({}^{E}\boldsymbol{\omega}_r^{A}\right)\times\boldsymbol{r}^{WJ}+{}^{E}\boldsymbol{\omega}_r^{A}\times\left({}^{E}\boldsymbol{\omega}^{A}\times\boldsymbol{r}^{WJ}\right) & for\quad r=7,8,\ldots,11\\ \frac{d}{dt}\left({}^{E}\boldsymbol{\omega}_{TFrl}^{A}\right)\times\boldsymbol{r}^{WJ}+{}^{E}\boldsymbol{\omega}_{TFrl}^{A}\times\left({}^{E}\boldsymbol{\omega}^{A}\times\boldsymbol{r}^{WJ}\right) & for\quad r=TFrl\\ 0 & otherwise\end{cases}$$

$$\frac{d}{dt}\left({}^{E}\boldsymbol{v}_t^{J}\right)=0$$

$$\frac{d}{dt}\left({}^{E}\boldsymbol{v}_{r}^{K}\right)=\frac{d}{dt}\left({}^{E}\boldsymbol{v}_{r}^{W}\right)+\begin{cases}{}^{E}\boldsymbol{\omega}_{r}^{A}\times\left({}^{E}\boldsymbol{\omega}^{A}\times\boldsymbol{r}^{WK}\right) & for \quad r=4,5,6\\ \frac{d}{dt}\left({}^{E}\boldsymbol{\omega}_{r}^{A}\right)\times\boldsymbol{r}^{WK}+{}^{E}\boldsymbol{\omega}_{r}^{A}\times\left({}^{E}\boldsymbol{\omega}^{A}\times\boldsymbol{r}^{WK}\right) & for \quad r=7,8,\ldots,11\\ \frac{d}{dt}\left({}^{E}\boldsymbol{\omega}_{TFrl}^{A}\right)\times\boldsymbol{r}^{WK}+{}^{E}\boldsymbol{\omega}_{TFrl}^{A}\times\left({}^{E}\boldsymbol{\omega}^{A}\times\boldsymbol{r}^{WK}\right) & for \quad r=TFrl\\ 0 & otherwise\end{cases}$$

$$\frac{d}{dt}\left({}^{E}\boldsymbol{v}_{t}^{K}\right)=0$$