

There are several points on a 2-bladed turbine: Z (platform reference), Y (platform mass center), T (tower node), O (tower-top / base-plate / yaw bearing mass center), U (nacelle mass center), V (arbitrary point on rotor-furl axis), W (arbitrary point on tail-furl axis), D (center of mass of structure that furls with the rotor [not including rotor]), IMU (nacelle inertial measurement unit), P (teeter pin), SG [shaft strain gage location: i.e., a point on the shaft a distance $ShiftGagL$ towards the nacelle from point P (or point Q for a 3-blader since point P does not exist)], Q (apex of coning angle), C (hub mass center), S1 (blade node for blade 1), S2 (blade node for blade 2), I (tail boom mass center), J (tail fin mass center), and K (tail fin center-of-pressure). There are also several reference frames: E (earth / inertial), X (platform / tower base), F (tower element body), B (tower-top / base plate), N (nacelle), R (structure that furls with the rotor—generator housing, etc...), L (low speed shaft on rotor end of LSS-compliance), H (hub / rotor), M1 (blade 1 element body), M2 (blade 2 element body), G (fixed in the high speed shaft / generator), and A (tail). The following are derivations of the position vectors, angular velocities, linear velocities, partial angular velocities, partial linear velocities, angular accelerations, and linear accelerations of all these points on the 2-bladed turbine (point SG's velocities and accelerations are not derived since they won't be used in the ensuing analysis). The velocities and accelerations of points on a 3-bladed turbine are very similar.

Position Vectors:

$$\mathbf{r}^Z = q_{Sg} \mathbf{z}_1 + q_{Hv} \mathbf{z}_2 - q_{Sw} \mathbf{z}_3 \quad (\text{relative to the ground})$$

$$\mathbf{r}^{ZY} = (PtfmRe f - PtfmCM) \mathbf{a}_2$$

$$\begin{aligned} \mathbf{r}^{ZT}(h) = & \left[\phi_1^{TFA}(h) q_{TFA1} + \phi_2^{TFA}(h) q_{TFA2} \right] \mathbf{a}_1 \\ & + \left\{ h + PtfmRe f - TwrDraft + TwrRBHt - \frac{1}{2} \left[S_{11}^{TFA}(h) q_{TFA1}^2 + S_{22}^{TFA}(h) q_{TFA2}^2 + 2S_{12}^{TFA}(h) q_{TFA1} q_{TFA2} \right. \right. \\ & \quad \left. \left. + S_{11}^{TSS}(h) q_{TSS1}^2 + S_{22}^{TSS}(h) q_{TSS2}^2 + 2S_{12}^{TSS}(h) q_{TSS1} q_{TSS2} \right] \right\} \mathbf{a}_2 \\ & + \left[\phi_1^{TSS}(h) q_{TSS1} + \phi_2^{TSS}(h) q_{TSS2} \right] \mathbf{a}_3 \end{aligned}$$

where,

$$S_{ij}^{TFA}(h) = \int_0^h \left[\frac{d\phi_i^{TFA}(h')}{dh'} \frac{d\phi_j^{TFA}(h')}{dh'} \right] dh' \quad (i, j = 1, 2) \quad (\text{which is symmetric})$$

and

$$S_{ij}^{TSS}(h) = \int_0^h \left[\frac{d\phi_i^{TSS}(h')}{dh'} \frac{d\phi_j^{TSS}(h')}{dh'} \right] dh' \quad (i, j = 1, 2) \quad (\text{which is symmetric})$$

Note limits on h : $0 \leq h \leq TowerHt + TwrDraft - TwrRBHt = TwrFlexL$

$$\begin{aligned} \mathbf{r}^{ZO} = & [q_{TFA1} + q_{TFA2}] \mathbf{a}_1 \\ & + \left\{ Ptfm\ Re\ f + TowerHt - \frac{1}{2} \left[\begin{aligned} & S_{11}^{TFA} (TwrFlexL) q_{TFA1}^2 + S_{22}^{TFA} (TwrFlexL) q_{TFA2}^2 + 2S_{12}^{TFA} (TwrFlexL) q_{TFA1} q_{TFA2} \\ & + S_{11}^{TSS} (TwrFlexL) q_{TSS1}^2 + S_{22}^{TSS} (TwrFlexL) q_{TSS2}^2 + 2S_{12}^{TSS} (TwrFlexL) q_{TSS1} q_{TSS2} \end{aligned} \right] \right\} \mathbf{a}_2 \\ & + [q_{TSS1} + q_{TSS2}] \mathbf{a}_3 \end{aligned}$$

$$\mathbf{r}^{OU} = NacCMxnd_1 + NacCMznd_2 - NacCMynd_3$$

$$\mathbf{r}^{OV} = RFrlPntxnd_1 + RFrlPntznd_2 - RFrlPntynd_3$$

$$\mathbf{r}^{VD} = (RFrlCMxn - RFrlPntxn) \mathbf{rf}_1 + (RFrlCMzn - RFrlPntzn) \mathbf{rf}_2 - (RFrlCMyn - RFrlPntyn) \mathbf{rf}_3$$

$$\mathbf{r}^{VIMU} = (NcIMUxn - RFrlPntxn) \mathbf{rf}_1 + (NcIMUzn - RFrlPntzn) \mathbf{rf}_2 - (NcIMUyn - RFrlPntyn) \mathbf{rf}_3$$

$$\mathbf{r}^{VP} = -RFrlPntxn \mathbf{rf}_1 + (Twr2Shft - RFrlPntzn) \mathbf{rf}_2 - (Yaw2Shft - RFrlPntyn) \mathbf{rf}_3 + OverHangc_1$$

$$\mathbf{r}^{PSG} = ShftGagLc_1$$

$$\mathbf{r}^{PQ} = -UndSlingg_1$$

$$\mathbf{r}^{QC} = HubCMg_1$$

$$\begin{aligned} \mathbf{r}^{QSI}(r) = & [\phi_1^{B1}(r) q_{B1F1} + \phi_2^{B1}(r) q_{B1F2} + \phi_3^{B1}(r) q_{B1E1}] \mathbf{j}_1^{B1} + [\psi_1^{B1}(r) q_{B1F1} + \psi_2^{B1}(r) q_{B1F2} + \psi_3^{B1}(r) q_{B1E1}] \mathbf{j}_2^{B1} \\ & + \left\{ r + HubRad - \frac{1}{2} [S_{11}^{B1}(r) q_{B1F1}^2 + S_{22}^{B1}(r) q_{B1F2}^2 + S_{33}^{B1}(r) q_{B1E1}^2 + 2S_{12}^{B1}(r) q_{B1F1} q_{B1F2} + 2S_{23}^{B1}(r) q_{B1F2} q_{B1E1} + 2S_{13}^{B1}(r) q_{B1F1} q_{B1E1}] \right\} \mathbf{j}_3^{B1} \end{aligned}$$

where,

$$S_{ij}^{B1}(r) = \int_0^r \left[\frac{d\phi_i^{B1}(r')}{dr'} \frac{d\phi_j^{B1}(r')}{dr'} + \frac{d\psi_i^{B1}(r')}{dr'} \frac{d\psi_j^{B1}(r')}{dr'} \right] dr' \quad (i, j = 1, 2, 3) \text{ (which is symmetric)}$$

and the twisted shape functions are defined as follows:

$$\begin{aligned} \phi_1^{B1}(r) &= \int_0^r \left\{ \int_0^{r'} \frac{d^2 \phi_1^{B1F}(r'')}{dr''^2} \cos[\theta_S^{B1}(r'')] dr'' \right\} dr', & \psi_1^{B1}(r) &= -\int_0^r \left\{ \int_0^{r'} \frac{d^2 \phi_1^{B1F}(r'')}{dr''^2} \sin[\theta_S^{B1}(r'')] dr'' \right\} dr', \\ \phi_2^{B1}(r) &= \int_0^r \left\{ \int_0^{r'} \frac{d^2 \phi_2^{B1F}(r'')}{dr''^2} \cos[\theta_S^{B1}(r'')] dr'' \right\} dr', & \psi_2^{B1}(r) &= -\int_0^r \left\{ \int_0^{r'} \frac{d^2 \phi_2^{B1F}(r'')}{dr''^2} \sin[\theta_S^{B1}(r'')] dr'' \right\} dr', \\ \phi_3^{B1}(r) &= \int_0^r \left\{ \int_0^{r'} \frac{d^2 \phi_1^{B1E}(r'')}{dr''^2} \sin[\theta_S^{B1}(r'')] dr'' \right\} dr', \text{ and } & \psi_3^{B1}(r) &= \int_0^r \left\{ \int_0^{r'} \frac{d^2 \phi_1^{B1E}(r'')}{dr''^2} \cos[\theta_S^{B1}(r'')] dr'' \right\} dr' \end{aligned}$$

The equation for $r^{QS2}(r)$ is similar.

Note limit on r : $0 \leq r \leq TipRad - HubRad = BldFlexL$

$$\mathbf{r}^{OW} = TFrlPntxnd_1 + TFrlPntznd_2 - TFrlPntynd_3$$

$$\mathbf{r}^{WI} = (BoomCMxn - TFrlPntxn) \mathbf{tf}_1 + (BoomCMzn - TFrlPntzn) \mathbf{tf}_2 - (BoomCMyn - TFrlPntyn) \mathbf{tf}_3$$

$$\mathbf{r}^{WJ} = (TFinCMxn - TFrlPntxn) \mathbf{tf}_1 + (TFinCMzn - TFrlPntzn) \mathbf{tf}_2 - (TFinCMyn - TFrlPntyn) \mathbf{tf}_3$$

$$\mathbf{r}^{WK} = (TFinCPxn - TFrlPntxn) \mathbf{tf}_1 + (TFinCPzn - TFrlPntzn) \mathbf{tf}_2 - (TFinCPyn - TFrlPntyn) \mathbf{tf}_3$$

Angular Velocities:

$${}^E\omega^X = \dot{q}_R \mathbf{z}_1 + \dot{q}_Y \mathbf{z}_2 - \dot{q}_P \mathbf{z}_3$$

$${}^E\omega^F(h) = {}^E\omega^X + \left[\frac{d\phi_1^{TSS}(h)}{dh} \dot{q}_{TSS1} + \frac{d\phi_2^{TSS}(h)}{dh} \dot{q}_{TSS2} \right] \mathbf{a}_1 - \left[\frac{d\phi_1^{TFA}(h)}{dh} \dot{q}_{TFA1} + \frac{d\phi_2^{TFA}(h)}{dh} \dot{q}_{TFA2} \right] \mathbf{a}_3$$

$${}^E\omega^B = {}^E\omega^X + \left[\frac{d\phi_1^{TSS}(h)}{dh} \bigg|_{h=TwrFlexL} \dot{q}_{TSS1} + \frac{d\phi_2^{TSS}(h)}{dh} \bigg|_{h=TwrFlexL} \dot{q}_{TSS2} \right] \mathbf{a}_1 - \left[\frac{d\phi_1^{TFA}(h)}{dh} \bigg|_{h=TwrFlexL} \dot{q}_{TFA1} + \frac{d\phi_2^{TFA}(h)}{dh} \bigg|_{h=TwrFlexL} \dot{q}_{TFA2} \right] \mathbf{a}_3$$

$${}^E\omega^N = {}^E\omega^B + \dot{q}_{Yaw} \mathbf{d}_2$$

$${}^E\omega^R = {}^E\omega^N + \dot{q}_{RFl} \mathbf{rfa}$$

where, $\mathbf{rfa} = \cos(RFlSkew) \cos(RFlTilt) \mathbf{d}_1 + \sin(RFlTilt) \mathbf{d}_2 - \sin(RFlSkew) \cos(RFlTilt) \mathbf{d}_3$

$${}^E\omega^L = {}^E\omega^R + \dot{q}_{DrTr} \mathbf{c}_1 + \dot{q}_{GeAz} \mathbf{c}_1$$

$${}^E\omega^H = {}^E\omega^L + \dot{q}_{Teet} \mathbf{f}_2$$

$${}^E\omega^{M1}(r) = {}^E\omega^H - \left[\frac{d\psi_1^{B1}(r)}{dr} \dot{q}_{B1F1} + \frac{d\psi_2^{B1}(r)}{dr} \dot{q}_{B1F2} + \frac{d\psi_3^{B1}(r)}{dr} \dot{q}_{B1E1} \right] \mathbf{j}_1^{B1} + \left[\frac{d\phi_1^{B1}(r)}{dr} \dot{q}_{B1F1} + \frac{d\phi_2^{B1}(r)}{dr} \dot{q}_{B1F2} + \frac{d\phi_3^{B1}(r)}{dr} \dot{q}_{B1E1} \right] \mathbf{j}_2^{B1}$$

The equation for ${}^E\omega^{M2}(r)$ is similar.

Since the generator is attached to the high speed shaft which may or may not rotate in the opposite direction of the low speed shaft and since q_{GeAz} represents the position of the low speed shaft near the entrance of the gearbox,

$${}^E\omega^G = {}^E\omega^R + GenDir \cdot GBRatio \cdot \dot{q}_{GeAz} \mathbf{c}_1$$

where,

$$GenDir = \begin{cases} -1 & \text{for } GBReverse = True \\ 1 & \text{for } GBReverse = False \end{cases}$$

$${}^E\boldsymbol{\omega}^A = {}^E\boldsymbol{\omega}^N + \dot{q}_{TFrl}\mathbf{tfa}$$

where, $\mathbf{tfa} = \cos(TFrlSkew)\cos(TFrlTilt)\mathbf{d}_1 + \sin(TFrlTilt)\mathbf{d}_2 - \sin(TFrlSkew)\cos(TFrlTilt)\mathbf{d}_3$

Linear Velocities:

$${}^E \mathbf{v}^Z = \dot{q}_{Sg} \mathbf{z}_1 + \dot{q}_{Hv} \mathbf{z}_2 - \dot{q}_{Sw} \mathbf{z}_3$$

$${}^E \mathbf{v}^Y = {}^E \mathbf{v}^Z + {}^E \boldsymbol{\omega}^X \times \mathbf{r}^{ZY}$$

$${}^E \mathbf{v}^T(h) = {}^E \mathbf{v}^Z + {}^X \mathbf{v}^T(h) + {}^E \boldsymbol{\omega}^X \times \mathbf{r}^{ZT}(h)$$

where,

$$\begin{aligned} {}^X \mathbf{v}^T(h) = & \left[\phi_1^{TFA}(h) \dot{q}_{TFA1} + \phi_2^{TFA}(h) \dot{q}_{TFA2} \right] \mathbf{a}_1 \\ & - \left[S_{11}^{TFA}(h) q_{TFA1} \dot{q}_{TFA1} + S_{22}^{TFA}(h) q_{TFA2} \dot{q}_{TFA2} + S_{12}^{TFA}(h) (\dot{q}_{TFA1} q_{TFA2} + q_{TFA1} \dot{q}_{TFA2}) \right. \\ & \quad \left. + S_{11}^{TSS}(h) q_{TSS1} \dot{q}_{TSS1} + S_{22}^{TSS}(h) q_{TSS2} \dot{q}_{TSS2} + S_{12}^{TSS}(h) (\dot{q}_{TSS1} q_{TSS2} + q_{TSS1} \dot{q}_{TSS2}) \right] \mathbf{a}_2 \\ & + \left[\phi_1^{TSS}(h) \dot{q}_{TSS1} + \phi_2^{TSS}(h) \dot{q}_{TSS2} \right] \mathbf{a}_3 \end{aligned}$$

$${}^E \mathbf{v}^O = {}^E \mathbf{v}^Z + {}^X \mathbf{v}^O + {}^E \boldsymbol{\omega}^X \times \mathbf{r}^{ZO}$$

where,

$$\begin{aligned} {}^X \mathbf{v}^O = & \left[\dot{q}_{TFA1} + \dot{q}_{TFA2} \right] \mathbf{a}_1 \\ & - \left[S_{11}^{TFA}(TwrFlexL) q_{TFA1} \dot{q}_{TFA1} + S_{22}^{TFA}(TwrFlexL) q_{TFA2} \dot{q}_{TFA2} + S_{12}^{TFA}(TwrFlexL) (\dot{q}_{TFA1} q_{TFA2} + q_{TFA1} \dot{q}_{TFA2}) \right. \\ & \quad \left. + S_{11}^{TSS}(TwrFlexL) q_{TSS1} \dot{q}_{TSS1} + S_{22}^{TSS}(TwrFlexL) q_{TSS2} \dot{q}_{TSS2} + S_{12}^{TSS}(TwrFlexL) (\dot{q}_{TSS1} q_{TSS2} + q_{TSS1} \dot{q}_{TSS2}) \right] \mathbf{a}_2 \\ & + \left[\dot{q}_{TSS1} + \dot{q}_{TSS2} \right] \mathbf{a}_3 \end{aligned}$$

$${}^E \mathbf{v}^U = {}^E \mathbf{v}^O + {}^E \boldsymbol{\omega}^N \times \mathbf{r}^{OU}$$

$${}^E \mathbf{v}^V = {}^E \mathbf{v}^O + {}^E \boldsymbol{\omega}^N \times \mathbf{r}^{OV}$$

$${}^E \mathbf{v}^D = {}^E \mathbf{v}^V + {}^E \boldsymbol{\omega}^R \times \mathbf{r}^{VD}$$

The equation for ${}^E \mathbf{v}^{IMU}$ is similar.

$${}^E \mathbf{v}^P = {}^E \mathbf{v}^V + {}^E \boldsymbol{\omega}^R \times \mathbf{r}^{VP}$$

$${}^E\mathbf{v}^Q = {}^E\mathbf{v}^P + {}^E\boldsymbol{\omega}^H \times \mathbf{r}^{PQ}$$

$${}^E\mathbf{v}^C = {}^E\mathbf{v}^Q + {}^E\boldsymbol{\omega}^H \times \mathbf{r}^{QC}$$

$${}^E\mathbf{v}^{SI}(r) = {}^E\mathbf{v}^Q + {}^H\mathbf{v}^{SI}(r) + {}^E\boldsymbol{\omega}^H \times \mathbf{r}^{QSI}(r)$$

where,

$$\begin{aligned} {}^H\mathbf{v}^{SI}(r) = & \left[\phi_1^{B1}(r)\dot{q}_{B1F1} + \phi_2^{B1}(r)\dot{q}_{B1F2} + \phi_3^{B1}(r)\dot{q}_{B1E1} \right] \mathbf{j}_1^{B1} + \left[\psi_1^{B1}(r)\dot{q}_{B1F1} + \psi_2^{B1}(r)\dot{q}_{B1F2} + \psi_3^{B1}(r)\dot{q}_{B1E1} \right] \mathbf{j}_2^{B1} \\ & - \left[\begin{aligned} & S_{11}^{B1}(r)q_{B1F1}\dot{q}_{B1F1} + S_{22}^{B1}(r)q_{B1F2}\dot{q}_{B1F2} + S_{33}^{B1}(r)q_{B1E1}\dot{q}_{B1E1} \\ & + S_{12}^{B1}(r)(\dot{q}_{B1F1}q_{B1F2} + q_{B1F1}\dot{q}_{B1F2}) + S_{23}^{B1}(r)(\dot{q}_{B1F2}q_{B1E1} + q_{B1F2}\dot{q}_{B1E1}) + S_{13}^{B1}(r)(\dot{q}_{B1F1}q_{B1E1} + q_{B1F1}\dot{q}_{B1E1}) \end{aligned} \right] \mathbf{j}_3^{B1} \end{aligned}$$

The equation for ${}^E\mathbf{v}^{S2}(r)$ is similar.

$${}^E\mathbf{v}^W = {}^E\mathbf{v}^O + {}^E\boldsymbol{\omega}^N \times \mathbf{r}^{OW}$$

$${}^E\mathbf{v}^I = {}^E\mathbf{v}^W + {}^E\boldsymbol{\omega}^A \times \mathbf{r}^{WI}$$

$${}^E\mathbf{v}^J = {}^E\mathbf{v}^W + {}^E\boldsymbol{\omega}^A \times \mathbf{r}^{WJ}$$

$${}^E\mathbf{v}^K = {}^E\mathbf{v}^W + {}^E\boldsymbol{\omega}^A \times \mathbf{r}^{WK}$$

Partial Angular Velocities:

Recall that: ${}^E\omega^{N_i}(\dot{q}, q, t) = \left(\sum_{r=1}^{22} {}^E\omega_r^{N_i}(q, t) \dot{q}_r \right) + {}^E\omega_i^{N_i}(q, t)$ for each rigid body N_i in the system. Note that all of the ${}^E\omega_i^{N_i}$ terms are zero as will be shown.

$${}^E\omega_r^X = \begin{cases} z_1 & \text{for } r = R \\ -z_3 & \text{for } r = P \\ z_2 & \text{for } r = Y \\ 0 & \text{otherwise} \end{cases}$$

$${}^E\omega_i^X = 0$$

$${}^E\omega_r^F(h) = {}^E\omega_r^X + \begin{cases} -\frac{d\phi_1^{TFA}(h)}{dh} a_3 & \text{for } r = TFA1 \\ \frac{d\phi_1^{TSS}(h)}{dh} a_1 & \text{for } r = TSS1 \\ -\frac{d\phi_2^{TFA}(h)}{dh} a_3 & \text{for } r = TFA2 \\ \frac{d\phi_2^{TSS}(h)}{dh} a_1 & \text{for } r = TSS2 \\ 0 & \text{otherwise} \end{cases}$$

$${}^E\omega_i^F(h) = 0$$

$${}^E\omega_r^B = {}^E\omega_r^X + \begin{cases} -\frac{d\phi_1^{TFA}(h)}{dh}\bigg|_{h=TwrFlexL} \mathbf{a}_3 & \text{for } r = TFA1 \\ \frac{d\phi_1^{TSS}(h)}{dh}\bigg|_{h=TwrFlexL} \mathbf{a}_1 & \text{for } r = TSS1 \\ -\frac{d\phi_2^{TFA}(h)}{dh}\bigg|_{h=TwrFlexL} \mathbf{a}_3 & \text{for } r = TFA2 \\ \frac{d\phi_2^{TSS}(h)}{dh}\bigg|_{h=TwrFlexL} \mathbf{a}_1 & \text{for } r = TSS2 \\ 0 & \text{otherwise} \end{cases}$$

$${}^E\omega_i^B = 0$$

$${}^E\omega_r^N = {}^E\omega_r^B + \begin{cases} \mathbf{d}_2 & \text{for } r = Yaw \\ 0 & \text{otherwise} \end{cases}$$

$${}^E\omega_i^N = 0$$

$${}^E\omega_r^R = {}^E\omega_r^N + \begin{cases} \mathbf{rfa} & \text{for } r = RFrl \\ 0 & \text{otherwise} \end{cases}$$

$${}^E\omega_i^R = 0$$

$${}^E\omega_r^L = {}^E\omega_r^R + \begin{cases} \mathbf{c}_1 & \text{for } r = GeAz \\ \mathbf{c}_1 & \text{for } r = DrTr \\ 0 & \text{otherwise} \end{cases}$$

$${}^E\omega_i^L = 0$$

$${}^E\omega_r^H = {}^E\omega_r^L + \begin{cases} f_2 & \text{for } r = Teet \\ 0 & \text{otherwise} \end{cases}$$

$${}^E\omega_i^H = 0$$

$${}^E\omega_r^{M1}(r) = {}^E\omega_r^H + \begin{cases} -\frac{d\psi_1^{B1}(r)}{dr} \mathbf{j}_1^{B1} + \frac{d\phi_1^{B1}(r)}{dr} \mathbf{j}_2^{B1} & \text{for } r = B1F1 \\ -\frac{d\psi_3^{B1}(r)}{dr} \mathbf{j}_1^{B1} + \frac{d\phi_3^{B1}(r)}{dr} \mathbf{j}_2^{B1} & \text{for } r = B1E1 \\ -\frac{d\psi_2^{B1}(r)}{dr} \mathbf{j}_1^{B1} + \frac{d\phi_2^{B1}(r)}{dr} \mathbf{j}_2^{B1} & \text{for } r = B1F2 \\ 0 & \text{otherwise} \end{cases}$$

$${}^E\omega_i^{M1}(r) = 0$$

The equations for ${}^E\omega_r^{M2}(r)$ and ${}^E\omega_i^{M2}(r)$ are similar.

$${}^E\omega_r^G = {}^E\omega_r^R + \begin{cases} GenDir \cdot GBRatioc_i & \text{for } r = GeAz \\ 0 & \text{otherwise} \end{cases}$$

$${}^E\omega_i^G = 0$$

$${}^E\omega_r^A = {}^E\omega_r^N + \begin{cases} tfa & \text{for } r = TFr1 \\ 0 & \text{otherwise} \end{cases}$$

$${}^E\omega_i^A = 0$$

Partial Linear Velocities:

Recall that: ${}^E \mathbf{v}^{X_i}(\dot{q}, q, t) = \left(\sum_{r=1}^{22} {}^E \mathbf{v}_r^{X_i}(q, t) \dot{q}_r \right) + {}^E \mathbf{v}_t^{X_i}(q, t)$ for each point X_i in the system. Note that all of the ${}^E \mathbf{v}_t^{X_i}$ terms are zero as will be shown.

$${}^E \mathbf{v}_r^Z = \begin{cases} \mathbf{z}_1 & \text{for } r = Sg \\ -\mathbf{z}_3 & \text{for } r = Sw \\ \mathbf{z}_2 & \text{for } r = Hv \\ 0 & \text{otherwise} \end{cases}$$

$${}^E \mathbf{v}_t^Z = 0$$

$${}^E \mathbf{v}_r^Y = {}^E \mathbf{v}_r^Z + \begin{cases} {}^E \boldsymbol{\omega}_r^X \times \mathbf{r}^{ZY} & \text{for } r = 4, 5, 6 \\ 0 & \text{otherwise} \end{cases}$$

$${}^E \mathbf{v}_t^Y = 0$$

$${}^E \mathbf{v}_r^T(h) = {}^E \mathbf{v}_r^Z + \begin{cases} {}^E \boldsymbol{\omega}_r^X \times \mathbf{r}^{ZT}(h) & \text{for } r = 4, 5, 6 \\ \phi_1^{TFA}(h) \mathbf{a}_1 - [S_{11}^{TFA}(h) q_{TFA1} + S_{12}^{TFA}(h) q_{TFA2}] \mathbf{a}_2 & \text{for } r = TFA1 \\ \phi_1^{TSS}(h) \mathbf{a}_3 - [S_{11}^{TSS}(h) q_{TSS1} + S_{12}^{TSS}(h) q_{TSS2}] \mathbf{a}_2 & \text{for } r = TSS1 \\ \phi_2^{TFA}(h) \mathbf{a}_1 - [S_{22}^{TFA}(h) q_{TFA2} + S_{12}^{TFA}(h) q_{TFA1}] \mathbf{a}_2 & \text{for } r = TFA2 \\ \phi_2^{TSS}(h) \mathbf{a}_3 - [S_{22}^{TSS}(h) q_{TSS2} + S_{12}^{TSS}(h) q_{TSS1}] \mathbf{a}_2 & \text{for } r = TSS2 \\ 0 & \text{otherwise} \end{cases}$$

$${}^E \mathbf{v}_t^T(h) = 0$$

$${}^E \mathbf{v}_r^O = {}^E \mathbf{v}_r^Z + \begin{cases} {}^E \boldsymbol{\omega}_r^X \times \mathbf{r}^{ZO} & \text{for } r = 4, 5, 6 \\ \mathbf{a}_1 - [S_{11}^{TFA} (TwrFlexL) q_{TFA1} + S_{12}^{TFA} (TwrFlexL) q_{TFA2}] \mathbf{a}_2 & \text{for } r = TFA1 \\ \mathbf{a}_3 - [S_{11}^{TSS} (TwrFlexL) q_{TSS1} + S_{12}^{TSS} (TwrFlexL) q_{TSS2}] \mathbf{a}_2 & \text{for } r = TSS1 \\ \mathbf{a}_1 - [S_{22}^{TFA} (TwrFlexL) q_{TFA2} + S_{12}^{TFA} (TwrFlexL) q_{TFA1}] \mathbf{a}_2 & \text{for } r = TFA2 \\ \mathbf{a}_3 - [S_{22}^{TSS} (TwrFlexL) q_{TSS2} + S_{12}^{TSS} (TwrFlexL) q_{TSS1}] \mathbf{a}_2 & \text{for } r = TSS2 \\ 0 & \text{otherwise} \end{cases}$$

$${}^E \mathbf{v}_t^O = 0$$

$${}^E \mathbf{v}_r^U = {}^E \mathbf{v}_r^O + \begin{cases} {}^E \boldsymbol{\omega}_r^N \times \mathbf{r}^{OU} & \text{for } r = 4, 5, \dots, 11 \\ 0 & \text{otherwise} \end{cases}$$

$${}^E \mathbf{v}_t^U = 0$$

$${}^E \mathbf{v}_r^V = {}^E \mathbf{v}_r^O + \begin{cases} {}^E \boldsymbol{\omega}_r^N \times \mathbf{r}^{OV} & \text{for } r = 4, 5, \dots, 11 \\ 0 & \text{otherwise} \end{cases}$$

$${}^E \mathbf{v}_t^V = 0$$

$${}^E \mathbf{v}_r^D = {}^E \mathbf{v}_r^V + \begin{cases} {}^E \boldsymbol{\omega}_r^R \times \mathbf{r}^{VD} & \text{for } r = 4, 5, \dots, 12 \\ 0 & \text{otherwise} \end{cases}$$

$${}^E \mathbf{v}_t^D = 0$$

The equations for ${}^E \mathbf{v}_r^{IMU}$ and ${}^E \mathbf{v}_t^{IMU}$ are similar.

$${}^E \mathbf{v}_r^P = {}^E \mathbf{v}_r^V + \begin{cases} {}^E \boldsymbol{\omega}_r^R \times \mathbf{r}^{VP} & \text{for } r = 4, 5, \dots, 12 \\ 0 & \text{otherwise} \end{cases}$$

$${}^E \mathbf{v}_t^P = 0$$

$${}^E \mathbf{v}_r^Q = {}^E \mathbf{v}_r^P + \begin{cases} {}^E \boldsymbol{\omega}_r^H \times \mathbf{r}^{PQ} & \text{for } r = 4, 5, \dots, 14 \\ {}^E \boldsymbol{\omega}_{Teet}^H \times \mathbf{r}^{PQ} & \text{for } r = Teet \\ 0 & \text{otherwise} \end{cases}$$

$${}^E \mathbf{v}_i^Q = 0$$

$${}^E \mathbf{v}_r^C = {}^E \mathbf{v}_r^Q + \begin{cases} {}^E \boldsymbol{\omega}_r^H \times \mathbf{r}^{QC} & \text{for } r = 4, 5, \dots, 14 \\ {}^E \boldsymbol{\omega}_{Teet}^H \times \mathbf{r}^{QC} & \text{for } r = Teet \\ 0 & \text{otherwise} \end{cases}$$

$${}^E \mathbf{v}_i^C = 0$$

$${}^E \mathbf{v}_r^{SI}(r) = {}^E \mathbf{v}_r^Q + \begin{cases} {}^E \boldsymbol{\omega}_r^H \times \mathbf{r}^{QSI}(r) & \text{for } r = 4, 5, \dots, 14 \\ \phi_1^{B1}(r) \mathbf{j}_1^{B1} + \psi_1^{B1}(r) \mathbf{j}_2^{B1} - [S_{11}^{B1}(r) q_{B1F1} + S_{12}^{B1}(r) q_{B1F2} + S_{13}^{B1}(r) q_{B1E1}] \mathbf{j}_3^{B1} & \text{for } r = B1F1 \\ \phi_3^{B1}(r) \mathbf{j}_1^{B1} + \psi_3^{B1}(r) \mathbf{j}_2^{B1} - [S_{33}^{B1}(r) q_{B1E1} + S_{23}^{B1}(r) q_{B1F2} + S_{13}^{B1}(r) q_{B1F1}] \mathbf{j}_3^{B1} & \text{for } r = B1E1 \\ \phi_2^{B1}(r) \mathbf{j}_1^{B1} + \psi_2^{B1}(r) \mathbf{j}_2^{B1} - [S_{22}^{B1}(r) q_{B1F2} + S_{12}^{B1}(r) q_{B1F1} + S_{23}^{B1}(r) q_{B1E1}] \mathbf{j}_3^{B1} & \text{for } r = B1F2 \\ {}^E \boldsymbol{\omega}_{Teet}^H \times \mathbf{r}^{QSI}(r) & \text{for } r = Teet \\ 0 & \text{otherwise} \end{cases}$$

$${}^E \mathbf{v}_i^{SI}(r) = 0$$

The equations for ${}^E \mathbf{v}_r^{S2}(r)$ and ${}^E \mathbf{v}_i^{S2}(r)$ are similar.

$${}^E \mathbf{v}_r^W = {}^E \mathbf{v}_r^O + \begin{cases} {}^E \boldsymbol{\omega}_r^N \times \mathbf{r}^{OW} & \text{for } r = 4, 5, \dots, 11 \\ 0 & \text{otherwise} \end{cases}$$

$${}^E \mathbf{v}_i^W = 0$$

$${}^E \mathbf{v}_r^I = {}^E \mathbf{v}_r^W + \begin{cases} {}^E \boldsymbol{\omega}_r^A \times \mathbf{r}^{WI} & \text{for } r = 4, 5, \dots, 11 \\ {}^E \boldsymbol{\omega}_{TFrl}^A \times \mathbf{r}^{WI} & \text{for } r = TFrI \\ 0 & \text{otherwise} \end{cases}$$

$${}^E \mathbf{v}_t^I = 0$$

$${}^E \mathbf{v}_r^J = {}^E \mathbf{v}_r^W + \begin{cases} {}^E \boldsymbol{\omega}_r^A \times \mathbf{r}^{WJ} & \text{for } r = 4, 5, \dots, 11 \\ {}^E \boldsymbol{\omega}_{TFrl}^A \times \mathbf{r}^{WJ} & \text{for } r = TFrI \\ 0 & \text{otherwise} \end{cases}$$

$${}^E \mathbf{v}_t^J = 0$$

$${}^E \mathbf{v}_r^K = {}^E \mathbf{v}_r^W + \begin{cases} {}^E \boldsymbol{\omega}_r^A \times \mathbf{r}^{WK} & \text{for } r = 4, 5, \dots, 11 \\ {}^E \boldsymbol{\omega}_{TFrl}^A \times \mathbf{r}^{WK} & \text{for } r = TFrI \\ 0 & \text{otherwise} \end{cases}$$

$${}^E \mathbf{v}_t^K = 0$$

Angular Accelerations:

Recall that: ${}^E\alpha^{N_i}(\ddot{q}, \dot{q}, q, t) = \left(\sum_{r=1}^{22} {}^E\omega_r^{N_i}(q, t) \ddot{q}_r \right) + \left[\sum_{r=1}^{22} \frac{d}{dt} ({}^E\omega_r^{N_i}(q, t)) \dot{q}_r \right] + \frac{d}{dt} ({}^E\omega_t^{N_i}(q, t))$ for each rigid body N_i in the system. Note that the $\frac{d}{dt} ({}^E\omega_r^{N_i})$ terms are all vector functions of $(\dot{q}, q, t)$ and that all of the $\frac{d}{dt} ({}^E\omega_t^{N_i})$ terms are zero as will be shown.

$$\frac{d}{dt} ({}^E\omega_r^X) = 0$$

$$\frac{d}{dt} ({}^E\omega_t^X) = 0$$

$$\frac{d}{dt} [{}^E\omega_r^F(h)] = \begin{cases} {}^E\omega^X \times {}^E\omega_r^F(h) & \text{for } r = 7, 8, \dots, 10 \\ 0 & \text{otherwise} \end{cases}$$

$$\frac{d}{dt} [{}^E\omega_t^F(h)] = 0$$

$$\frac{d}{dt} ({}^E\omega_r^B) = \begin{cases} {}^E\omega^X \times {}^E\omega_r^B & \text{for } r = 7, 8, \dots, 10 \\ 0 & \text{otherwise} \end{cases}$$

$$\frac{d}{dt} ({}^E\omega_t^B) = 0$$

$$\frac{d}{dt} ({}^E\omega_r^N) = \frac{d}{dt} ({}^E\omega_r^B) + \begin{cases} {}^E\omega^B \times {}^E\omega_{Yaw}^N & \text{for } r = Yaw \\ 0 & \text{otherwise} \end{cases}$$

$$\frac{d}{dt} ({}^E\omega_t^N) = 0$$

$$\frac{d}{dt}({}^E\omega_r^R) = \frac{d}{dt}({}^E\omega_r^N) + \begin{cases} {}^E\omega^N \times {}^E\omega_{RFrl}^R & \text{for } r = RFrl \\ 0 & \text{otherwise} \end{cases}$$

$$\frac{d}{dt}({}^E\omega_i^R) = 0$$

$$\frac{d}{dt}({}^E\omega_r^L) = \frac{d}{dt}({}^E\omega_r^R) + \begin{cases} {}^E\omega^R \times {}^E\omega_{GeAz}^L & \text{for } r = GeAz \\ {}^E\omega^R \times {}^E\omega_{DrTr}^L & \text{for } r = DrTr \\ 0 & \text{otherwise} \end{cases}$$

$$\frac{d}{dt}({}^E\omega_i^L) = 0$$

$$\frac{d}{dt}({}^E\omega_r^H) = \frac{d}{dt}({}^E\omega_r^L) + \begin{cases} {}^E\omega^H \times {}^E\omega_{Teet}^H & \text{for } r = Teet \\ 0 & \text{otherwise} \end{cases}$$

$$\frac{d}{dt}({}^E\omega_i^H) = 0$$

$$\frac{d}{dt}[{}^E\omega_r^{M1}(r)] = \frac{d}{dt}({}^E\omega_r^H) + \begin{cases} {}^E\omega^H \times {}^E\omega_{B1F1}^{M1}(r) & \text{for } r = B1F1 \\ {}^E\omega^H \times {}^E\omega_{B1E1}^{M1}(r) & \text{for } r = B1E1 \\ {}^E\omega^H \times {}^E\omega_{B1F2}^{M1}(r) & \text{for } r = B1F2 \\ 0 & \text{otherwise} \end{cases}$$

$$\frac{d}{dt}[{}^E\omega_i^{M1}(r)] = 0$$

The equations for $\frac{d}{dt}[{}^E\omega_r^{M2}(r)]$ and $\frac{d}{dt}[{}^E\omega_i^{M2}(r)]$ are similar.

$$\frac{d}{dt}({}^E\omega_r^G) = \frac{d}{dt}({}^E\omega_r^R) + \begin{cases} {}^E\omega^R \times {}^E\omega_{GeAz}^G & \text{for } r = GeAz \\ 0 & \text{otherwise} \end{cases}$$

$$\frac{d}{dt}({}^E\omega_i^G) = 0$$

$$\frac{d}{dt}({}^E\omega_r^A) = \frac{d}{dt}({}^E\omega_r^N) + \begin{cases} {}^E\omega^N \times {}^E\omega_{TFrl}^A & \text{for } r = TFrl \\ 0 & \text{otherwise} \end{cases}$$

$$\frac{d}{dt}({}^E\omega_i^A) = 0$$

Linear Accelerations:

Recall that: ${}^E \mathbf{a}^{X_i}(\ddot{q}, \dot{q}, q, t) = \left(\sum_{r=1}^{22} {}^E \mathbf{v}_r^{X_i}(q, t) \ddot{q}_r \right) + \left[\sum_{r=1}^{22} \frac{d}{dt}({}^E \mathbf{v}_r^{X_i}(q, t)) \dot{q}_r \right] + \frac{d}{dt}({}^E \mathbf{v}_t^{X_i}(q, t))$ for each point X_i in the system. Note that the $\frac{d}{dt}({}^E \mathbf{v}_r^{X_i})$ terms are all vector functions of $(\dot{q}, q, t)$ and that all of the $\frac{d}{dt}({}^E \mathbf{v}_t^{X_i})$ terms are zero as will be shown.

$$\frac{d}{dt}({}^E \mathbf{v}_r^Z) = 0$$

$$\frac{d}{dt}({}^E \mathbf{v}_t^Z) = 0$$

$$\frac{d}{dt}({}^E \mathbf{v}_r^Y) = \begin{cases} {}^E \boldsymbol{\omega}_r^X \times ({}^E \boldsymbol{\omega}^X \times \mathbf{r}^{ZY}) & \text{for } r = 4, 5, 6 \\ 0 & \text{otherwise} \end{cases}$$

$$\frac{d}{dt}({}^E \mathbf{v}_t^Y) = 0$$

$$\frac{d}{dt}({}^E \mathbf{v}_r^T(h)) = \begin{cases} {}^E \boldsymbol{\omega}_r^X \times [{}^X \mathbf{v}^T(h) + {}^E \boldsymbol{\omega}^X \times \mathbf{r}^{ZT}(h)] & \text{for } r = 4, 5, 6 \\ -[S_{11}^{TFA}(h) \dot{q}_{TFA1} + S_{12}^{TFA}(h) \dot{q}_{TFA1}] \mathbf{a}_2 + {}^E \boldsymbol{\omega}^X \times {}^E \mathbf{v}_{TFA1}^T(h) & \text{for } r = TFA1 \\ -[S_{11}^{TSS}(h) \dot{q}_{TSS1} + S_{12}^{TSS}(h) \dot{q}_{TSS2}] \mathbf{a}_2 + {}^E \boldsymbol{\omega}^X \times {}^E \mathbf{v}_{TSS1}^T(h) & \text{for } r = TSS1 \\ -[S_{22}^{TFA}(h) \dot{q}_{TFA2} + S_{12}^{TFA}(h) \dot{q}_{TFA1}] \mathbf{a}_2 + {}^E \boldsymbol{\omega}^X \times {}^E \mathbf{v}_{TFA2}^T(h) & \text{for } r = TFA2 \\ -[S_{22}^{TSS}(h) \dot{q}_{TSS2} + S_{12}^{TSS}(h) \dot{q}_{TSS1}] \mathbf{a}_2 + {}^E \boldsymbol{\omega}^X \times {}^E \mathbf{v}_{TSS2}^T(h) & \text{for } r = TSS2 \\ 0 & \text{otherwise} \end{cases}$$

$$\frac{d}{dt}({}^E \mathbf{v}_t^T(h)) = 0$$

$$\frac{d}{dt}({}^E \mathbf{v}_r^O) = \begin{cases} {}^E \boldsymbol{\omega}_r^X \times ({}^X \mathbf{v}^O + {}^E \boldsymbol{\omega}^X \times \mathbf{r}^{ZO}) & \text{for } r = 4, 5, 6 \\ -[S_{11}^{TFA}(TwrFlexL)\dot{q}_{TFA1} + S_{12}^{TFA}(TwrFlexL)\dot{q}_{TFA1}] \mathbf{a}_2 + {}^E \boldsymbol{\omega}^X \times {}^E \mathbf{v}_{TFA1}^O & \text{for } r = TFA1 \\ -[S_{11}^{TSS}(TwrFlexL)\dot{q}_{TSS1} + S_{12}^{TSS}(TwrFlexL)\dot{q}_{TSS2}] \mathbf{a}_2 + {}^E \boldsymbol{\omega}^X \times {}^E \mathbf{v}_{TSS1}^O & \text{for } r = TSS1 \\ -[S_{22}^{TFA}(TwrFlexL)\dot{q}_{TFA2} + S_{12}^{TFA}(TwrFlexL)\dot{q}_{TFA1}] \mathbf{a}_2 + {}^E \boldsymbol{\omega}^X \times {}^E \mathbf{v}_{TFA2}^O & \text{for } r = TFA2 \\ -[S_{22}^{TSS}(TwrFlexL)\dot{q}_{TSS2} + S_{12}^{TSS}(TwrFlexL)\dot{q}_{TSS1}] \mathbf{a}_2 + {}^E \boldsymbol{\omega}^X \times {}^E \mathbf{v}_{TSS2}^O & \text{for } r = TSS2 \\ 0 & \text{otherwise} \end{cases}$$

$$\frac{d}{dt}({}^E \mathbf{v}_t^O) = 0$$

$$\frac{d}{dt}({}^E \mathbf{v}_r^U) = \frac{d}{dt}({}^E \mathbf{v}_r^O) + \begin{cases} {}^E \boldsymbol{\omega}_r^N \times ({}^E \boldsymbol{\omega}^N \times \mathbf{r}^{OU}) & \text{for } r = 4, 5, 6 \\ \frac{d}{dt}({}^E \boldsymbol{\omega}_r^N) \times \mathbf{r}^{OU} + {}^E \boldsymbol{\omega}_r^N \times ({}^E \boldsymbol{\omega}^N \times \mathbf{r}^{OU}) & \text{for } r = 7, 8, \dots, 11 \\ 0 & \text{otherwise} \end{cases}$$

$$\frac{d}{dt}({}^E \mathbf{v}_t^U) = 0$$

$$\frac{d}{dt}({}^E \mathbf{v}_r^V) = \frac{d}{dt}({}^E \mathbf{v}_r^O) + \begin{cases} {}^E \boldsymbol{\omega}_r^N \times ({}^E \boldsymbol{\omega}^N \times \mathbf{r}^{OV}) & \text{for } r = 4, 5, 6 \\ \frac{d}{dt}({}^E \boldsymbol{\omega}_r^N) \times \mathbf{r}^{OV} + {}^E \boldsymbol{\omega}_r^N \times ({}^E \boldsymbol{\omega}^N \times \mathbf{r}^{OV}) & \text{for } r = 7, 8, \dots, 11 \\ 0 & \text{otherwise} \end{cases}$$

$$\frac{d}{dt}({}^E \mathbf{v}_t^V) = 0$$

$$\frac{d}{dt}({}^E \mathbf{v}_r^D) = \frac{d}{dt}({}^E \mathbf{v}_r^V) + \begin{cases} {}^E \boldsymbol{\omega}_r^R \times ({}^E \boldsymbol{\omega}^R \times \mathbf{r}^{VD}) & \text{for } r = 4, 5, 6 \\ \frac{d}{dt}({}^E \boldsymbol{\omega}_r^R) \times \mathbf{r}^{VD} + {}^E \boldsymbol{\omega}_r^R \times ({}^E \boldsymbol{\omega}^R \times \mathbf{r}^{VD}) & \text{for } r = 7, 8, \dots, 12 \\ 0 & \text{otherwise} \end{cases}$$

$$\frac{d}{dt}({}^E \mathbf{v}_t^D) = 0$$

The equations for $\frac{d}{dt}({}^E \mathbf{v}_r^{IMU})$ and $\frac{d}{dt}({}^E \mathbf{v}_t^{IMU})$ are similar.

$$\frac{d}{dt}({}^E \mathbf{v}_r^P) = \frac{d}{dt}({}^E \mathbf{v}_r^V) + \begin{cases} {}^E \boldsymbol{\omega}_r^R \times ({}^E \boldsymbol{\omega}^R \times \mathbf{r}^{VP}) & \text{for } r = 4, 5, 6 \\ \frac{d}{dt}({}^E \boldsymbol{\omega}_r^R) \times \mathbf{r}^{VP} + {}^E \boldsymbol{\omega}_r^R \times ({}^E \boldsymbol{\omega}^R \times \mathbf{r}^{VP}) & \text{for } r = 7, 8, \dots, 12 \\ 0 & \text{otherwise} \end{cases}$$

$$\frac{d}{dt}({}^E \mathbf{v}_t^P) = 0$$

$$\frac{d}{dt}({}^E \mathbf{v}_r^Q) = \frac{d}{dt}({}^E \mathbf{v}_r^P) + \begin{cases} {}^E \boldsymbol{\omega}_r^H \times ({}^E \boldsymbol{\omega}^H \times \mathbf{r}^{PQ}) & \text{for } r = 4, 5, 6 \\ \frac{d}{dt}({}^E \boldsymbol{\omega}_r^H) \times \mathbf{r}^{PQ} + {}^E \boldsymbol{\omega}_r^H \times ({}^E \boldsymbol{\omega}^H \times \mathbf{r}^{PQ}) & \text{for } r = 7, 8, \dots, 14 \\ \frac{d}{dt}({}^E \boldsymbol{\omega}_{Teet}^H) \times \mathbf{r}^{PQ} + {}^E \boldsymbol{\omega}_{Teet}^H \times ({}^E \boldsymbol{\omega}^H \times \mathbf{r}^{PQ}) & \text{for } r = Teet \\ 0 & \text{otherwise} \end{cases}$$

$$\frac{d}{dt}({}^E \mathbf{v}_t^Q) = 0$$

$$\frac{d}{dt}({}^E \mathbf{v}_r^C) = \frac{d}{dt}({}^E \mathbf{v}_r^Q) + \begin{cases} {}^E \boldsymbol{\omega}_r^H \times ({}^E \boldsymbol{\omega}^H \times \mathbf{r}^{QC}) & \text{for } r = 4, 5, 6 \\ \frac{d}{dt}({}^E \boldsymbol{\omega}_r^H) \times \mathbf{r}^{QC} + {}^E \boldsymbol{\omega}_r^H \times ({}^E \boldsymbol{\omega}^H \times \mathbf{r}^{QC}) & \text{for } r = 7, 8, \dots, 14 \\ \frac{d}{dt}({}^E \boldsymbol{\omega}_{Teet}^H) \times \mathbf{r}^{QC} + {}^E \boldsymbol{\omega}_{Teet}^H \times ({}^E \boldsymbol{\omega}^H \times \mathbf{r}^{QC}) & \text{for } r = Teet \\ 0 & \text{otherwise} \end{cases}$$

$$\frac{d}{dt}({}^E \mathbf{v}_t^C) = 0$$

$$\frac{d}{dt}[{}^E \mathbf{v}_r^{SI}(r)] = \frac{d}{dt}({}^E \mathbf{v}_r^Q) + \begin{cases} {}^E \boldsymbol{\omega}_r^H \times [{}^H \mathbf{v}^{SI}(r) + {}^E \boldsymbol{\omega}^H \times \mathbf{r}^{QSI}(r)] & \text{for } r = 4, 5, 6 \\ \frac{d}{dt}({}^E \boldsymbol{\omega}_r^H) \times \mathbf{r}^{QSI}(r) + {}^E \boldsymbol{\omega}_r^H \times [{}^H \mathbf{v}^{SI}(r) + {}^E \boldsymbol{\omega}^H \times \mathbf{r}^{QSI}(r)] & \text{for } r = 7, 8, \dots, 14 \\ -[S_{11}^{BI}(r) \dot{q}_{B1F1} + S_{12}^{BI}(r) \dot{q}_{B1F2} + S_{13}^{BI}(r) \dot{q}_{B1E1}] \mathbf{j}_3^{BI} + {}^E \boldsymbol{\omega}^H \times {}^E \mathbf{v}_{B1F1}^{SI}(r) & \text{for } r = B1F1 \\ -[S_{33}^{BI}(r) \dot{q}_{B1E1} + S_{23}^{BI}(r) \dot{q}_{B1F2} + S_{13}^{BI}(r) \dot{q}_{B1F1}] \mathbf{j}_3^{BI} + {}^E \boldsymbol{\omega}^H \times {}^E \mathbf{v}_{B1E1}^{SI}(r) & \text{for } r = B1E1 \\ -[S_{22}^{BI}(r) \dot{q}_{B1F2} + S_{12}^{BI}(r) \dot{q}_{B1F1} + S_{23}^{BI}(r) \dot{q}_{B1E1}] \mathbf{j}_3^{BI} + {}^E \boldsymbol{\omega}^H \times {}^E \mathbf{v}_{B1F2}^{SI}(r) & \text{for } r = B1F2 \\ \frac{d}{dt}({}^E \boldsymbol{\omega}_{Teet}^H) \times \mathbf{r}^{QSI}(r) + {}^E \boldsymbol{\omega}_{Teet}^H \times [{}^H \mathbf{v}^{SI}(r) + {}^E \boldsymbol{\omega}^H \times \mathbf{r}^{QSI}(r)] & \text{for } r = Teet \\ 0 & \text{otherwise} \end{cases}$$

$$\frac{d}{dt}[{}^E \mathbf{v}_t^{SI}(r)] = 0$$

The equations for $\frac{d}{dt}[{}^E \mathbf{v}_r^{S2}(r)]$ and $\frac{d}{dt}[{}^E \mathbf{v}_t^{S2}(r)]$ are similar.

$$\frac{d}{dt}({}^E \mathbf{v}_r^W) = \frac{d}{dt}({}^E \mathbf{v}_r^O) + \begin{cases} {}^E \boldsymbol{\omega}_r^N \times ({}^E \boldsymbol{\omega}^N \times \mathbf{r}^{OW}) & \text{for } r = 4, 5, 6 \\ \frac{d}{dt}({}^E \boldsymbol{\omega}_r^N) \times \mathbf{r}^{OW} + {}^E \boldsymbol{\omega}_r^N \times ({}^E \boldsymbol{\omega}^N \times \mathbf{r}^{OW}) & \text{for } r = 7, 8, \dots, 11 \\ 0 & \text{otherwise} \end{cases}$$

$$\frac{d}{dt}({}^E \mathbf{v}_t^W) = 0$$

$$\frac{d}{dt}({}^E \mathbf{v}_r^I) = \frac{d}{dt}({}^E \mathbf{v}_r^W) + \begin{cases} {}^E \boldsymbol{\omega}_r^A \times ({}^E \boldsymbol{\omega}^A \times \mathbf{r}^{WI}) & \text{for } r = 4, 5, 6 \\ \frac{d}{dt}({}^E \boldsymbol{\omega}_r^A) \times \mathbf{r}^{WI} + {}^E \boldsymbol{\omega}_r^A \times ({}^E \boldsymbol{\omega}^A \times \mathbf{r}^{WI}) & \text{for } r = 7, 8, \dots, 11 \\ \frac{d}{dt}({}^E \boldsymbol{\omega}_{TFrl}^A) \times \mathbf{r}^{WI} + {}^E \boldsymbol{\omega}_{TFrl}^A \times ({}^E \boldsymbol{\omega}^A \times \mathbf{r}^{WI}) & \text{for } r = TFrI \\ 0 & \text{otherwise} \end{cases}$$

$$\frac{d}{dt}({}^E \mathbf{v}_t^I) = 0$$

$$\frac{d}{dt}({}^E \mathbf{v}_r^J) = \frac{d}{dt}({}^E \mathbf{v}_r^W) + \begin{cases} {}^E \boldsymbol{\omega}_r^A \times ({}^E \boldsymbol{\omega}^A \times \mathbf{r}^{WJ}) & \text{for } r = 4, 5, 6 \\ \frac{d}{dt}({}^E \boldsymbol{\omega}_r^A) \times \mathbf{r}^{WJ} + {}^E \boldsymbol{\omega}_r^A \times ({}^E \boldsymbol{\omega}^A \times \mathbf{r}^{WJ}) & \text{for } r = 7, 8, \dots, 11 \\ \frac{d}{dt}({}^E \boldsymbol{\omega}_{TFrl}^A) \times \mathbf{r}^{WJ} + {}^E \boldsymbol{\omega}_{TFrl}^A \times ({}^E \boldsymbol{\omega}^A \times \mathbf{r}^{WJ}) & \text{for } r = TFrJ \\ 0 & \text{otherwise} \end{cases}$$

$$\frac{d}{dt}({}^E \mathbf{v}_t^J) = 0$$

$$\frac{d}{dt}({}^E\mathbf{v}_r^K) = \frac{d}{dt}({}^E\mathbf{v}_r^W) + \begin{cases} {}^E\boldsymbol{\omega}_r^A \times ({}^E\boldsymbol{\omega}^A \times \mathbf{r}^{WK}) & \text{for } r = 4, 5, 6 \\ \frac{d}{dt}({}^E\boldsymbol{\omega}_r^A) \times \mathbf{r}^{WK} + {}^E\boldsymbol{\omega}_r^A \times ({}^E\boldsymbol{\omega}^A \times \mathbf{r}^{WK}) & \text{for } r = 7, 8, \dots, 11 \\ \frac{d}{dt}({}^E\boldsymbol{\omega}_{TFrI}^A) \times \mathbf{r}^{WK} + {}^E\boldsymbol{\omega}_{TFrI}^A \times ({}^E\boldsymbol{\omega}^A \times \mathbf{r}^{WK}) & \text{for } r = TFrI \\ 0 & \text{otherwise} \end{cases}$$

$$\frac{d}{dt}({}^E\mathbf{v}_t^K) = 0$$